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RELATIVITY: THE SPECIAL AND GENERAL THEORY

BY ALBERT EINSTEIN

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PREFACE

(December, 1916)

The present book is intended, as far as possible, to give an exact insight into the theory of Relativity to those readers who, from a general scientific and philosophical point of view, are interested in the theory, but who are not conversant with the mathematical apparatus of theoretical physics. The work presumes a standard of education corresponding to that of a university matriculation examination, and, despite the shortness of the book, a fair amount of patience and force of will on the part of the reader. The author has spared himself no pains in his endeavour to present the main ideas in the simplest and most intelligible form, and on the whole, in the sequence and connection in which they actually originated. In the interest of clearness, it appeared to me inevitable that I should repeat myself frequently, without paying the slightest attention to the elegance of the presentation. I adhered scrupulously to the precept of that brilliant theoretical physicist L. Boltzmann, according to whom matters of elegance ought to be left to the tailor and to the cobbler. I make no pretence of having withheld from the reader difficulties which are inherent to the subject. On the other hand, I have purposely treated the empirical physical foundations of the theory in a "step-motherly" fashion, so that readers unfamiliar with physics may not feel like the wanderer who was unable to see the forest for the trees. May the book bring someone a few happy hours of suggestive thought!

December, 1916
A. EINSTEIN

前言

（1916 年 12 月）

本书旨在尽可能准确地把相对论介绍给如下这些读者：他们基于普通的科学和哲学观点对相对论感兴趣，但不熟悉理论物理的数学工具。该书假定读者具有了普通大学预科的教育水平。尽管本书很短，但还是需要读者有足够的耐心和意愿。作者不厌其烦地以最简单和最容易理解的方式展示了其主要思想，并且总的来说，保持了这些思想原有的顺序和联系。为了表达清晰，我不得不经常重复，毫不在意表达是否优雅。我一丝不苟地遵守了杰出理论物理学家 L • 玻尔兹曼的那条戒律：优雅的事情应该留给裁缝和鞋匠。我并不假装对读者隐瞒了本话

题固有的困难。另一方面，我刻意以“继母”的方式来处理该理论的经验物理基础，以便让不熟悉物理的读者不觉得自己是一个因为树木而看不见森林的流浪者。愿这本书能带给你几个小时的快乐思想时光！

1916 年 12 月
A.爱因斯坦

PART I

THE SPECIAL THEORY OF RELATIVITY

PHYSICAL MEANING OF GEOMETRICAL PROPOSITIONS

第一部分

狭义相对论

01. 几何命题的物理意义

In your schooldays most of you who read this book made acquaintance with the noble building of Euclid's geometry, and you remember -- perhaps with more respect than love -- the magnificent structure, on the lofty staircase of which you were chased about for uncounted hours by conscientious teachers. By reason of our past experience, you would certainly regard everyone with disdain who should pronounce even the most out-of-the-way proposition of this science to be untrue. But perhaps this feeling of proud certainty would leave you immediately if someone were to ask you: "What, then, do you mean by the assertion that these propositions are true?" Let us proceed to give this question a little consideration.

大多数读这本书的人，都在你们的学生时代接触过欧几里得几何学的高贵建筑。你应该还记得它的宏伟结构，（也许是尊重多于热爱），你还被尽职尽责的老师们在其高高的楼梯上追赶了无数个小时。根据我们过去的经验，如果谁敢说这门科学里哪怕最偏僻的命题是错的，您肯定会鄙视他。但如果有人问你："那么，你所说的命题为真是什么意思？"，也许你的这种自豪的确定感就会立刻消失。现在就让我们来给这个问题一点小小的思考。

Geometry sets out form certain conceptions such as "plane," "point," and "straight line," with which we are able to associate more or less definite ideas, and from certain simple propositions (axioms) which, in virtue of these ideas, we are inclined to accept as "true." Then, on the basis of a logical process, the justification of which we feel ourselves compelled to admit, all remaining propositions are shown to follow from those axioms, i.e. they are proven. A proposition is then correct ("true") when it has been derived in the recognized manner from the axioms. The question of "truth" of the individual geometrical propositions is thus reduced to one of the "truth" of the axioms. Now it has long been known that the last question is not only unanswerable by the methods of geometry, but that it is in itself entirely without meaning. We cannot ask whether it is true that only one straight line goes through two points. We can only say that Euclidean geometry deals with things called "straight lines," to each of which is ascribed the property of being uniquely determined by two points situated on it. The concept "true" does not tally with the assertions of pure geometry, because by the word "true" we are eventually in the habit of designating always the correspondence with a "real" object; geometry, however, is not concerned with the relation of the ideas involved in it to objects of experience, but only with the logical connection of these ideas among themselves.

几何学是从一些概念和简单的命题（公理）开始的。像平面、点、直线这些概念，我们可以或多或少地将其与确切的含义联系在一起。基于这些公理明显的正确性，我们倾向于接受它们为“真理”。然后，基于逻辑推理，我们不得不承认，所有剩余的命题都是从这些公理得出来的，也就是说，它们得到了证明。如果一个命题可以从这些公理以公认的方式推导出来，它就是正确的。所以一个几何命题的正确性问题就归结为几何公理的正确性问题。我们早就知道，几何公理的正确性不但是无法用几何方法证明的，而且其本身也是完全没有意义的。我们不能问过两点只能做一条直线是否正确。我们只能说欧几里得几何处理称为“直线”的东西，每条直线都可由位于其上的两个点唯一确定。“正确”这个概念与纯几何的断言不符，因为对于“正确”这个词，我们习惯于把它和“真实”的事物联系在一起；然而几何学并不在意它与生活经验中的物体的联系，而只关心它自己那些主意之间的逻辑关系。

It is not difficult to understand why, in spite of this, we feel constrained to call the propositions of geometry "true." Geometrical ideas correspond to more or less exact objects in nature, and these last are undoubtedly the exclusive cause of the genesis of those ideas. Geometry ought to refrain from such a course, in order to give to its structure the largest possible logical unity. The practice, for example, of seeing in a "distance" two marked positions on a practically rigid body is something which is lodged deeply in our habit of thought. We are accustomed further to regard three points as being situated on a straight line, if their apparent positions can be made to coincide for observation with one eye, under suitable choice of our place of observation.

不难理解为什么，尽管如此，我们仍然还得将几何命题称为“真”。几何学的概念或多或少地对应于自然界中的确切对象，而这些对象无疑是那些几何概念的起源的唯一起因。为了保证其结构的最大逻辑统一，几何学应该远离这样的起因。像从一个现实中刚体上的两点间看见“距离”这种现象，是深植于我们心中的思维习惯。还有，如果我们从某个合适的观察地方，可以用一只眼睛看到三个点的位置是重合的，我们就认为它们位于同一条直线上。

If, in pursuance of our habit of thought, we now supplement the propositions of Euclidean geometry by the single proposition that two points on a practically rigid body always correspond to the same distance (line-interval), independently of any changes in position to which we may subject the body, the propositions of Euclidean geometry then resolve themselves into propositions on the possible relative position of practically rigid bodies.* Geometry which has been supplemented in this way is then to be treated as a branch of physics. We can now legitimately ask as to the "truth" of geometrical propositions interpreted in this way, since we are justified in asking whether these propositions are satisfied for those real things we have associated with the geometrical ideas. In less exact terms we can express this by saying that by the "truth" of a geometrical proposition in this sense we understand its validity for a construction with rule and compasses.

如果按照我们的思维习惯，我们现在给欧几里得几何的命题做一个补充：现实刚体上的两点总是对应于相同的距离（线间隔），这个距离与该刚体的位置变化毫无关系。这样，欧几里得几何的命题就转化为关于现实世界刚体相对位置的命题。* 以这种方式加以补充的几何学则可以被视为物理学的一个分支。我们现在就可以合法地追问以这种方式解释的几何命题的“正确性”，因为我们有理由来追问这些与几何概念联系起来的那些真实事物的命题是否得到了满足。用不太准确的话来说，当我们在这种意义上说一个几何命题为“真”时，我们知道它是可以用尺规作图法构造出来的。

Of course the conviction of the "truth" of geometrical propositions in this sense is founded exclusively on rather incomplete experience. For the present we shall assume the "truth" of the geometrical propositions, then at a later stage (in the general theory of relativity) we shall see that this "truth" is limited, and we shall consider the extent of its limitation.

当然，对于在这种意义下的几何命题的“正确性”的信念是建立在相当不完整的经验之上的。现在我们先假设这些几何命题的“正确性”，然后在稍后的阶段（在广义相对论中）我们会看到这个“正确性”是有局限的，我们再考虑这种局限的程度。

Notes

*) It follows that a natural object is associated also with a straight line. Three points A, B and C on a rigid body thus lie in a straight line when the points A and C being given, B is chosen such that the sum of the distances AB and BC is as short as possible. This

incomplete suggestion will suffice for the present purpose.

注

*）据此，一个自然对象也与直线关联起来。刚体上的三个点 A、B 和 C 位于同一条直线上，意味着当给定点 A 和 C 时，点 B 要保证距离 AB 和 BC 的总和最短。这个不完整的表述对于目前的目的是足够了。

THE SYSTEM OF CO-ORDINATES

02．坐标系

On the basis of the physical interpretation of distance which has been indicated, we are also in a position to establish the distance between two points on a rigid body by means of measurements. For this purpose we require a " distance " (rod S) which is to be used once and for all, and which we employ as a standard measure. If, now, A and B are two points on a rigid body, we can construct the line joining them according to the rules of geometry ; then, starting from A, we can mark off the distance S time after time until we reach B. The number of these operations required is the numerical measure of the distance AB. This is the basis of all measurement of length. *

基于目前我们对距离的物理意义的理解，我们也可以通过测量来确定刚体上两点之间的距离。为此，我们需要一劳永逸地使用一个“距离”（杆 S），将其作为度量标准。如果现在 A 和 B 是刚体上的两个点，我们可以根据几何学的规则构造连接它们的线，那么，从 A 开始，我们可以通过一次又一次地标记，来量出整个距离 S，直到我们到达 B。所需的这样操作的次数就是距离 AB 的数值度量。这就是所有长度测量的基础。*

Every description of the scene of an event or of the position of an object in space is based on the specification of the point on a rigid body (body of reference) with which that event or object coincides. This applies not only to scientific description, but also to everyday life. If I analyse the place specification " Times Square, New York," **A I arrive at the following result. The earth is the rigid body to which the specification of place refers; " Times Square, New York," is a well-defined point, to which a name has been assigned, and with which the event coincides in space.**B

每一个对事件场景或物体位置的描述，都基于对一个正好与事件或物体重合的刚体上的点的具体描述。这不仅适用于科学描述，也适用于日常生活。如果让我来分析“纽约时代广场”这个对地点的描述，**A 我可以得出以下结果。地球就是该地点描述所参照的刚体，“纽约时代广场”是一个明确定义了的点，并已被赋予了名称，并且该点与发生的事件在空间重合。**B

This primitive method of place specification deals only with places on the surface of rigid bodies, and is dependent on the existence of points on this surface which are distinguishable from each other. But we can free ourselves from both of these limitations without altering the nature of our specification of position. If, for instance, a cloud is hovering over Times Square, then we can determine its position relative to the surface of the earth by erecting a pole perpendicularly on the Square, so that it reaches the cloud. The length of the pole measured with the standard measuring-rod, combined with the specification of the position of the foot of the pole, supplies us with a complete place specification. On the basis of this illustration, we are able to see the manner in which a refinement of the conception of position has been developed.

这种原始的确定位置的方法只能处理刚体表面上的位置，它依赖于这个事实：该表面上存在这些点，并且它们之间可以相互区分。但我们可以摆脱这两种限制，而不改变位置描述的性质。比如，如果一片云飘荡在时代广场上空，我们可以这样确定它相对于地球表面的位置：竖起一根垂直于广场的杆子，并让它直达云端。用标准测量杆量出该杆子的长度，再加上该杆脚的位置，我们就得到了对该片云彩的完整位置描述。通过这个例子，我们可以看出可以如何对位置描述进行改进：

(a) We imagine the rigid body, to which the place specification is referred, supplemented in such a manner that the object whose position we require is reached by. the completed rigid body.

（a）想象我们位置描述所参照的刚体得到了充分的延展，它能够完全覆盖我们需要描述的物体的位置。

(b) In locating the position of the object, we make use of a number (here the length of the pole measured with the measuring-rod) instead of designated points of reference.

(b) 在确定物体的位置时，我们用一个数字（这里是用测量杆测量的杆子的长度）来代替相应的参考点。

(c) We speak of the height of the cloud even when the pole which reaches the cloud has not been erected. By means of optical observations of the cloud from different positions on the ground, and taking into account the properties of the propagation of light, we determine the length of the pole we should have required in order to reach the cloud.

(c) 即便是直达云端的杆子没竖起来，我们还是可以给出云的高度。通过从地面不同位置对云彩进行光学观测，再利用光的传播特性，我们就可以确定我们达到云端所需要的杆子的长度。

From this consideration we see that it will be advantageous if, in the description of position, it should be possible by means of numerical measures to make ourselves independent of the existence of marked positions (possessing names) on the rigid body of reference. In the physics of measurement this is attained by the application of the Cartesian system of co-ordinates.

从以上过程可以看出，在描述位置时，如果我们能用度量的数值来代替参照刚体上的刻度（每一刻度都有具体的名称），可以方便很多。在物理学里，这些度量可以通过笛卡尔坐标系来实现。

This consists of three plane surfaces perpendicular to each other and rigidly attached to a rigid body. Referred to a system of co-ordinates, the scene of any event will be determined (for the main part) by the specification of the lengths of the three perpendiculars or co-ordinates (x, y, z) which can be dropped from the scene of the event to those three plane surfaces. The lengths of these three perpendiculars can be determined by a series of manipulations with rigid measuring-rods performed according to the rules and methods laid down by Euclidean geometry.

它由三个相互垂直，并且刚性地附着在一个刚体上的平面组成。在一个坐标系中，任何事件的场景（主要部分）都可以由三个从事物到这三个平面的垂线的长度或叫坐标(x, y, z)来确定。这三个垂线的长度可以按照欧几里得几何的规则，通过一系列刚性量杆的操作来确定。

In practice, the rigid surfaces which constitute the system of co-ordinates are generally not available ; furthermore, the magnitudes of the co-ordinates are not actually determined by constructions with rigid rods, but by indirect means. If the results of physics and astronomy are to maintain their clearness, the physical meaning of specifications of position must always be sought in accordance with the above considerations. ***

实际上，构成坐标系的刚性表面通常并不存在。另外，坐标的刻度也不是用刚性杆量出来的，而是用间接方式得到的。如果要保证物理和天文学的结果有一个清晰的含义，在考虑位置描述的物理意义的时候，就必须明白这些情况。 ***

We thus obtain the following result: Every description of events in space involves the use of a rigid body to which such events have to be referred. The resulting relationship takes for granted that the laws of Euclidean geometry hold for "distances;" the "distance" being represented physically by means of the convention of two marks on a rigid body.

因此我们可以得到以下结果：每一个对空间事物的描述，都必须使用一个刚体作参考系。这种关系自然而然地认为欧几里得几何的规则适用于“距离”，而这个“距离”则可用刚体上的两个标记来实际代表。

Notes

* Here we have assumed that there is nothing left over i.e. that the measurement gives a whole number. This difficulty is got over by the use of divided measuring-rods, the introduction of which does not demand any fundamentally new method.

注

* 这里我们假设没有任何剩余，也就是测量值是一个整数。通过把测量杆细分，我们可以克服这个困难，而这种细分并不需要引入任何全新的方法。

**A Einstein used "Potsdamer Platz, Berlin" in the original text. In the authorised translation this was supplemented with "Tranfalgar Square, London". We have changed this to "Times Square, New York", as this is the most well known/identifiable location to English speakers in the present day. [Note by the janitor.]

**A 爱因斯坦在原文中使用了“柏林波茨坦广场”。在授权翻译中，它被替换为“伦敦特拉法加广场”。我们已将其更改为“纽约时代广场”，因为对于今天说英语的人来说，这是最知名/最容易识别的地方。[看门人的笔记。]

**B It is not necessary here to investigate further the significance of the expression "coincidence in space." This conception is sufficiently obvious to ensure that differences of opinion are scarcely likely to arise as to its applicability in practice.

**B 这里没有必要进一步调查“空间重合”这种表达的重要性。这个概念已经足够明显了，在实际应用中极少会造成歧义。

*** A refinement and modification of these views does not become necessary until we come to deal with the general theory of relativity, treated in the second part of this book.

*** 对这种观念的细化和修改现在还不必要，等我们处理广义相对论时（在本书的第二部分）再来进行。

SPACE AND TIME IN CLASSICAL MECHANICS

03. 经典力学中的空间和时间

The purpose of mechanics is to describe how bodies change their position in space with "time." I should load my conscience with grave sins against the sacred spirit of lucidity were I to formulate the aims of mechanics in this way, without serious reflection and detailed explanations. Let us proceed to disclose these sins.

力学的目的是描述物体如何随“时间”改变它们在空间的位置。如果我就这样描述力学的目的，而不给予其认真的反思和详细的解释，我的良心会在神圣的清晰灵魂前充满沉重的负罪感。接下来就让我们来揭露这些罪恶。

It is not clear what is to be understood here by "position" and "space." I stand at the window of a railway carriage which is travelling uniformly, and drop a stone on the embankment, without throwing it. Then, disregarding the influence of the air resistance, I see the stone descend in a straight line. A pedestrian who observes the misdeed from the footpath notices that the stone falls to earth in a parabolic curve. I now ask: Do the "positions" traversed by the stone lie "in reality" on a straight line or on a parabola? Moreover, what is meant here by motion "in space" ? From the considerations of the previous section the answer is self-evident. In the first place we entirely shun the vague word "space," of which, we must honestly acknowledge, we cannot form the slightest conception, and we replace it by "motion relative to a practically rigid body of reference." The positions relative to the body of reference (railway carriage or embankment) have already been defined in detail in the preceding section. If instead of " body of reference " we insert " system of co-ordinates," which is a useful idea for mathematical description, we are in a position to say : The stone traverses a straight line relative to a system of co-ordinates rigidly attached to the carriage, but relative to a system of co-ordinates rigidly attached to the ground (embankment) it describes a parabola. With the aid of this example it is clearly seen that there is no such thing as an independently existing trajectory (lit. "path-curve"*), but only a trajectory relative to a particular body of reference.

这里的“位置”和“空间”的意义并不清楚。比如说，我站在一个正在匀速行驶的火车车厢的窗户前，往路堤上丢下一块石头（让石头自由下落，而不是把它扔出去）。如果不考虑空气阻力的影响，我会看到石头成直线下降。一个在人行道上的行人，如果他注意到了这件坏事，会看到石头沿着抛物线掉到了地上。我现在想问：在现实中，石头的下落的“位置”是在一条直线上还是在一条抛物线上？还有，这里的“在空间中的”运动是什么意思？根据上一节的解释，这里的答案是不言而喻的。首先我们应该完全避免含糊不清的“空间”一词，因为我们必须老实地承认，我们对其并没有丝毫的概念，我们把它替换为“相对于于一个实际参照刚体的运动”。这些相对于参考系的位置（火车车厢或路堤）已经在前面一节详细定义过了。如果我们把“参考系”换成在数学描述中有用的“坐标系”，我们就可以这样说：对于一个连接到车厢的坐标系，石头走的是一条直线，而对于一个连接到地面（路堤）的坐标系，石头的轨迹则是一条抛物线。通过这个例子，我们可以清楚地看到，根本就没有完全独立存在的轨迹（“路径曲线”*）这回事，一个轨迹总是相对于某个特定的参考系的。

In order to have a complete description of the motion, we must specify how the body alters its position with time ; i.e. for every point on the trajectory it must be stated at what time the body is situated there. These data must be supplemented by such a definition of time that, in virtue of this definition, these time-values can be regarded essentially as magnitudes (results of measurements) capable of observation. If we take our stand on the ground of classical mechanics, we can satisfy this requirement for our illustration in the following manner. We imagine two clocks of identical construction ; the man at the railway-carriage window is holding one of them, and the man on the footpath the other. Each of the observers determines the position on his own reference-body occupied by the stone at each tick of the clock he is holding in his hand. In this connection we have not taken account of the inaccuracy involved by the finiteness of the velocity of propagation of light. With this and with a second difficulty prevailing here we shall have to deal in detail later.

为了对运动有一个完整的描述，我们必须指定身体如何随时间改变其位置；即对于每个点必须说明身体所处时间的轨迹那里。这些数据必须辅以这样的时间定义根据这个定义，这些时间值可以被视为基本上作为量级（测量结果）能够观察。如果我们站在经典的立场上力学，我们可以满足我们在插图中的这个要求以下方式。我们想象两个结构相同的时钟；火车车厢窗口的那个人拿着其中一个，而人行道上的另一个人。每个观察者决定每次滴答时石头在他自己的参考体上的位置他手里拿着的时钟。在这方面我们没有考虑到有限性所涉及的不准确性光的传播速度。有了这个和第二个这里普遍存在的困难，我们将不得不在稍后详细处理。

Notes

*) That is, a curve along which the body moves.

注：

*） 即物体沿其运动的曲线。

THE GALILEIAN SYSTEM OF CO-ORDINATES

04．伽利略坐标系

As is well known, the fundamental law of the mechanics of Galilei-Newton, which is known as the law of inertia, can be stated thus: A body removed sufficiently far from other bodies continues in a state of rest or of uniform motion in a straight line. This law not only says something about the motion of the bodies, but it also indicates the reference-bodies or systems of coordinates, permissible in mechanics, which can be used in mechanical description. The visible fixed stars are bodies for which the law of inertia certainly holds to a high degree of approximation. Now if we use a system of co-ordinates which is rigidly attached to the earth, then, relative to this system, every fixed star describes a circle of immense radius in the course of an astronomical day, a result which is opposed to the statement of the law of inertia. So that if we adhere to this law we must refer these motions only to systems of coordinates relative to which the fixed stars do not move in a circle. A system of co-ordinates of which the state of motion is such that the law of inertia holds relative to it is called a " Galilean system of co-ordinates." The laws of the mechanics of Galilei-Newton can be regarded as valid only for a Galilean system of co-ordinates.

众所周知，伽利略-牛顿力学的基本定律，也即我们所称的惯性定律，可以表述为：一个与其他物体相距足够远的物体会保持静止状态或匀速直线运动状态。该定律不仅描述了物体的运动，也指出了参考物或坐标系，该坐标系可以用来描述动力学。可见的恒星在很大程度上符合惯性定律。现在如果我们使用一个固定在地球上的坐标系，那么，相对于这个系统，经过每一个天文日，每个恒星都画出一个半径巨大的圆，这样的结果与惯性定律相反。因此，如果要遵守这项定律，我们必须这样选择坐标系，使得恒星相对于它们不做圆周运动。如果一个坐标系能让惯性定律在其中成立，我们就称之为“伽利略坐标系”。伽利略-牛顿力学定律仅对伽利略坐标系成立。

THE PRINCIPLE OF RELATIVITY
(IN THE RESTRICTED SENSE)

05. 相对性原理（狭义的）

In order to attain the greatest possible clearness, let us return to our example of the railway carriage supposed to be travelling uniformly. We call its motion a uniform translation ("uniform" because it is of constant velocity and direction, " translation " because although the carriage changes its position relative to the embankment yet it does not rotate in so doing). Let us imagine a raven flying through the air in such a manner that its motion, as observed from the embankment, is uniform and in a straight line. If we were to observe the flying raven from the moving railway carriage. we should find that the motion of the raven would be one of different velocity and direction, but that it would still be uniform and in a straight line. Expressed in an abstract manner we may say : If a mass m is moving uniformly in a straight line with respect to a co-ordinate system K, then it will also be moving uniformly and in a straight line relative to a second co-ordinate system K1 provided that the latter is executing a uniform translatory motion with respect to K. In accordance with the discussion contained in the preceding section, it follows that:

为了最明白地说明问题，让我们回到我们匀速行驶的火车车厢的例子。我们称它的运动为匀速平移（“匀速”是因为它具有恒定的速度和方向，“平移”是因为尽管车厢相对于路堤改变了位置，但它并未旋转）。让我们想象一只乌鸦在空中飞翔，从路堤来看，它的运动匀速直线运动。如果我们从移动的火车车厢里观察该乌鸦，我们会发现乌鸦飞行的速度及方向都不同，但它的运动仍然是匀速的并在一条直线上。用抽象的方式表达，我们可以说：如果一个质量 m 正在相对于坐标系 K 做匀速直线运动，那么它相对于坐标系 K1 也会做匀速直线运动到，前提是是后者（K1)相对于 K 在做匀速平移运动。根据上一节中的讨论，我们可以得出如下结论：

If K is a Galilean co-ordinate system. then every other co-ordinate system K' is a Galilean one, when, in relation to K, it is in a condition of uniform motion of translation. Relative to K1 the mechanical laws of Galilei-Newton hold good exactly as they do with respect to K.

如果 K 是一个伽利略坐标系，那么任何一个与之做匀速平移运动的坐标系 K1 也是伽利略系统。伽利略-牛顿的力学定律完全适用于 K1，就像它们适用于坐标系 K 一样。

We advance a step farther in our generalisation when we express the tenet thus: If, relative to K, K1 is a uniformly moving co-ordinate system devoid of rotation, then natural phenomena run their course with respect to K1 according to exactly the same general laws as with respect to K. This statement is called the principle of relativity (in the restricted sense).

更进一步，我们可以把其概括为：如果相对于 K，K1 是一个匀速运动并且没有旋转的坐标系，那么任何自然现象就会在 K 和 K1 中遵从完全相同的定律。这个表述就是相对性原理（狭义的）。

As long as one was convinced that all natural phenomena were capable of representation with the help of classical mechanics, there was no need to doubt the validity of this principle of relativity. But in view of the more recent development of electrodynamics and optics it became more and more evident that classical mechanics affords an insufficient foundation for the physical description of all natural phenomena. At this juncture the question of the validity of the principle of relativity became ripe for discussion, and it did not appear impossible that the answer to this question might be in the negative.

只要你相信所有自然现象都能够用经典力学来表示，就无需怀疑这个相对性原理的有效性。但鉴于电动力学和光学的最新发展，现在越来越明显的是，经典力学不足以为描述所有的自然想象提供一个基础。此时此刻，相对性原理的有效性问题的讨论时机已经成熟，而且对这个问题的否定答案也并非不可能。

Nevertheless, there are two general facts which at the outset speak very much in favour of the validity of the principle of relativity. Even though classical mechanics does not supply us with a sufficiently broad basis for the theoretical presentation of all physical phenomena, still we must grant it a considerable measure of " truth," since it supplies us with the actual motions of the heavenly bodies with a delicacy of detail little short of wonderful. The principle of relativity must therefore apply with great accuracy in the domain of mechanics. But that a principle of such broad generality should hold with such exactness in one domain of phenomena, and yet should be invalid for another, is a priori not very probable.

然而，有两个普遍的事实从一开始就支持相对性原理的有效性。尽管经典力学没有为所有物理现象提供足够广泛的基础，我们仍然必须给予它相当大的“真实”分量，因为它对天体实际运动的描述非常细致，特别精彩。因此相对性原理肯定能非常准确地应用于力学。一个如此广泛的一般性原则能如此精确地适用于一个领域的现象，却在另一领域则无效，是不太可能的。

We now proceed to the second argument, to which, moreover, we shall return later. If the principle of relativity (in the restricted sense) does not hold, then the Galilean co-ordinate systems K, K1, K2, etc., which are moving uniformly relative to each other, will not be equivalent for the description of natural phenomena. In this case we should be constrained to believe that natural laws are capable of being formulated in a particularly simple manner, and of course only on condition that, from amongst all possible Galilean co-ordinate systems, we should have chosen one (K[0]) of a particular state of motion as our body of reference. We should then be justified (because of its merits for the description of natural phenomena) in calling this system " absolutely at rest," and all other Galilean systems K " in motion." If, for instance, our embankment were the system K[0] then our railway carriage would be a system K, relative to which less simple laws would hold than with respect to K[0]. This diminished simplicity would be due to the fact that the carriage K would be in motion (i.e."really")with respect to K[0]. In the general laws of nature which have been formulated with reference to K, the magnitude and direction of the velocity of the carriage would necessarily play a part. We should expect, for instance, that the note emitted by an organ pipe placed with its axis parallel to the direction of travel would be different from that emitted if the axis of the pipe were placed perpendicular to this direction.

我们现在来讲第二个论点，另外，我们后面还会返回来继续讨论。如果相对性原理（狭义上的）不成立，则相对于彼此做匀速直线运动的伽利略坐标系 K、K1、K2 等对自然现象的表达就不会等价。如果情形确实如此，我们应该能用一个特别简单的方式来描述自然定律，但前提是，在所有可能的伽利略坐标系中，我们能选择一个处于特定运动状态的(K[0])作为我们的参考系。鉴于该参考系在描述自然现象上的优点，我们有理由把它当作是“绝对静止的”，而所有其他伽利略系统 K 都在做“运动”。例如，如果我们的路堤是系统 K[0]，那么我们的火车车厢将是一个系统 K，对它适用的定律不会像在 K[0]中那么简单。之所以简单性在这里减少了，是因为车厢 K 在相对于 K[0]做（真正的）运动。在相对于 K 建立的通用自然定律中，车厢速度的大小和方向必然会起作用。例如，对于同一个管风琴，当其轴线平行于其前进方向时，和当其轴线垂直于其前进方向时，所发出的音符应该不同。

Now in virtue of its motion in an orbit round the sun, our earth is comparable with a railway carriage travelling with a velocity of about 30 kilometres per second. If the principle of relativity were not valid we should therefore expect that the direction of motion of the earth at any moment would enter into the laws of nature, and also that physical systems in their behaviour would be dependent on the orientation in space with respect to the earth. For owing to the alteration in direction of the velocity of revolution of the earth in the course of a year, the earth cannot be at rest relative to the hypothetical system K[0] throughout the whole year. However, the most careful observations have never revealed such anisotropic properties in terrestrial physical space, i.e. a physical non-equivalence of different directions. This is very powerful argument in favour of the principle of relativity.

现在来考虑地球。由于它在围绕太阳运动，我们可以把它当成是以大约每秒 30 公里的速度行驶的火车车厢。如果相对性原理不成立，我们会预期地球运动的方向在自然规律中会有所表达，而且物理系统的行为将取决于其相对于地球的空间方向。由于地球在一年中的转动速度的方向是改变的，地球相对于假设的系统 K[0]不能全年都处于静止状态。然而，最仔细的观察也从未发现过这种地面物理空间的各向异性

的特性，即不同方向的物理不等价。这是对相对性原理的一个非常有力的支持论据。

THE THEOREM OF THE
ADDITION OF VELOCITIES
EMPLOYED IN CLASSICAL MECHANICS
06. 经典力学中的速度相加定理

Let us suppose our old friend the railway carriage to be travelling along the rails with a constant velocity v, and that a man traverses the length of the carriage in the direction of travel with a velocity w. How quickly or, in other words, with what velocity W does the man advance relative to the embankment during the process ? The only possible answer seems to result from the following consideration: If the man were to stand still for a second, he would advance relative to the embankment through a distance v equal numerically to the velocity of the carriage. As a consequence of his walking, however, he traverses an additional distance w relative to the carriage, and hence also relative to the embankment, in this second, the distance w being numerically equal to the velocity with which he is walking. Thus in total he covers the distance W=v+w relative to the embankment in the second considered. We shall see later that this result, which expresses the theorem of the addition of velocities employed in classical mechanics, cannot be maintained ; in other words, the law that we have just written down does not hold in reality. For the time being, however, we shall assume its correctness.

让我们假设我们的老朋友火车车厢正在以恒定速度 v 沿着轨道行驶，车厢里的一个人正在以速度 w 沿相同方向行进。那么人相对于路堤的速度 W 是多少？唯一可能的答案似乎来自以下考虑：如果他相对于车厢保持不动，那么经过一秒钟，他会相对于路堤前进距离 v。但是因为他在行进，他在一秒内相对于车厢前进了额外的距离 w，因此同样相对于路堤，他也多移动了距离 w。因此在一秒内，他相对于路堤总共移动了距离 W=v+w。后面我们将会看到这个结果，即经典力学的速度相加定理，是无法保持的。换句话说，我们刚刚写下的定律在现实中是不成立的。不过现在呢，我们还是暂时假设它是正确的。

THE APPARENT INCOMPATIBILITY OF THE

LAW OF PROPAGATION OF LIGHT WITH THE PRINCIPLE OF RELATIVITY

07. 光的传播定律与相对性原理的明显不兼容

There is hardly a simpler law in physics than that according to which light is propagated in empty space. Every child at school knows, or believes he knows, that this propagation takes place in straight lines with a velocity c= 300,000 km./sec. At all events we know with great exactness that this velocity is the same for all colours, because if this were not the case, the minimum of emission would not be observed simultaneously for different colours during the eclipse of a fixed star by its dark neighbour. By means of similar considerations based on observations of double stars, the Dutch astronomer De Sitter was also able to show that the velocity of propagation of light cannot depend on the velocity of motion of the body emitting the light. The assumption that this velocity of propagation is dependent on the direction "in space" is in itself improbable.

在物理学里，几乎没有比光在真空中的传播规律更简单的定律了。学校里的每个孩子都知道，或者觉得他知道，光以 c = 300,000 公里/秒的速度沿直线传播。我们都非常了解，在任何情况下，这个速度对于所有颜色都适用，因为如果不是这样，当一个恒星被它临近较暗的恒星遮挡时，我们就不会同时观测到它发出的那些微弱的颜色。基于类似的考虑和对双星的观测结果，荷兰天文学家德西特得出结论：光的传播速度不可能取决于发光物体的运动速度。传播速度取决于“在空间”的方向这种假设本身也是不可能的。

In short, let us assume that the simple law of the constancy of the velocity of light c (in vacuum) is justifiably believed by the child at school. Who would imagine that this simple law has plunged the conscientiously thoughtful physicist into the greatest intellectual difficulties? Let us consider how these difficulties arise.

简而言之，我们可以认为光（在真空中）具有恒定速度这样一个简单定律是学校里的每个孩子都相信的。谁能想到，这个简单的定律却让认真思考的物理学家遇到了最大的认知困难？让我们看看这些困难是如何产生的。

Of course we must refer the process of the propagation of light (and indeed every other process) to a rigid reference-body (co-ordinate system). As such a system let us again choose our embankment. We shall imagine the air above it to have been removed. If a ray of light be sent along the embankment, we see from the above that the tip of the ray will be transmitted with the velocity c relative to the embankment. Now let us suppose that our railway carriage is again travelling along the railway lines with the velocity v, and that its direction is the same as that of the ray of light, but its velocity of course much less. Let us inquire about the velocity of propagation of the ray of light relative to the carriage. It is obvious that we can here apply the consideration of the previous section, since the ray of light plays the part of the man walking along relatively to the carriage. The velocity w of the man relative to the embankment is here replaced by the velocity of light relative to the embankment. w is the required velocity of light with respect to the carriage, and we have

当然，我们必须为光的传播过程（实际上为每一个过程）选定一个参考系（坐标系）。我们这里还是用我们的路堤作为这样的坐标系。我们想象一下它上面的空气已经被去除了。如果沿着路堤发出一束光，我们会从上面看到光束的前端以速度 c 相对于路堤传播。现在再假设我们的火车车厢又是以速度 v 沿铁路线行驶，并且它的方向与光束相同，当然该速度（相对于光速）要小很多。我们想知道光束相对于车厢的速度是多少。很明显我们可以这里应用上一节的考虑，因为这里的光线扮演了在车厢里行走的人的角色。在这里，人相对于路堤的速度 w 由光相对于路堤的速度所代替。 w 是光相对于车厢的速度，其公式是

$$w = c-v.$$

The velocity of propagation of a ray of light relative to the carriage thus comes cut smaller than c.

所以，光线相对于车厢的传播速度变得比 c 小了。

But this result comes into conflict with the principle of relativity set forth in Section V. For, like every other general law of nature, the law of the transmission of light in vacuo [in vacuum] must, according to the principle of relativity, be the same for the railway carriage as reference-body as when the rails are the body of reference. But, from our above consideration, this would appear to be impossible. If every ray of light is propagated relative to the embankment with the velocity c, then for this reason it would appear that another law of propagation of light must necessarily hold with respect to the carriage -- a result contradictory to the principle of relativity.

但这个结果与第 V 节中所讲的相对性原理相冲突。因为，就像其他所有的自然定律一样，根据相对性原理，光在真空中的传输定律[在真空

中]必须是相同的，不管是用车厢作为参考物，还是用路堤作为参考物。但是，从我们上面的讨论看，这似乎是不可能的。如果每一束光都相对于都相对于路堤以速度 c 传播，那么就会有另外一个相对于车厢的光的传播定律，而这是于相对性原理相矛盾的。

In view of this dilemma there appears to be nothing else for it than to abandon either the principle of relativity or the simple law of the propagation of light in vacuo. Those of you who have carefully followed the preceding discussion are almost sure to expect that we should retain the principle of relativity, which appeals so convincingly to the intellect because it is so natural and simple. The law of the propagation of light in vacuo would then have to be replaced by a more complicated law conformable to the principle of relativity. The development of theoretical physics shows, however, that we cannot pursue this course. The epoch-making theoretical investigations of H. A. Lorentz on the electrodynamical and optical phenomena connected with moving bodies show that experience in this domain leads conclusively to a theory of electromagnetic phenomena, of which the law of the constancy of the velocity of light in vacuo is a necessary consequence. Prominent theoretical physicists were therefore more inclined to reject the principle of relativity, in spite of the fact that no empirical data had been found which were contradictory to this principle.

鉴于这种困境，似乎没有别的办法了，要么放弃相对性原理，要么放弃光在真空中的传播中一简单定律。如果你认真听取了我们前面的讨论，几乎可以肯定你会希望我们保留相对性原理，因为它是如此自然和简单，所以在思想上更具有吸引力。这样以来，光在真空中的传播定律必须用更复杂的定律来表达，以便符合相对性原则。然而，理论物理学的发展表明，我们不能走这条路。在电动力学和光学上，H.A.洛伦兹做了一些与运动有关的划时代的理论研究，并得出了关于电磁现象的决定性的结果，而真空中的光速不变是其结论之一。因此，著名的理论物理学家更倾向于拒绝相对性原理，尽管没有发现任何实验数据与这个原则相矛盾。

At this juncture the theory of relativity entered the arena. As a result of an analysis of the physical conceptions of time and space, it became evident that in reality there is not the least incompatibility between the principle of relativity and the law of propagation of light, and that by systematically holding fast to both these laws a logically rigid theory could be arrived at. This theory has been called the special theory of relativity to distinguish it from the extended theory, with which we shall deal later. In the following pages we shall present the fundamental ideas of the special theory of relativity.

就在这时，相对论进入了舞台。作为一个对时间和空间的物理概念进行分析的结果，很明显，在现实中相对性原理与光的传播定律没有丝毫的不相容，并且通过系统地严格遵守这两个定律，可以得出一个逻辑严密的理论。这个理论现在被称为狭义相对论，以区别于我们后面要讲的扩展理论。在下面几页里，我们将介绍狭义相对论的基本思想。

ON THE IDEA OF TIME IN PHYSICS

08. 物理学中的时间概念

Lightning has struck the rails on our railway embankment at two places A and B far distant from each other. I make the additional assertion that these two lightning flashes occurred simultaneously. If I ask you whether there is sense in this statement, you will answer my question with a decided "Yes." But if I now approach you with the request to explain to me the sense of the statement more precisely, you find after some consideration that the answer to this question is not so easy as it appears at first sight.

闪电击中了我们铁轨上两个相距很远的地方 A 和 B，我还说那两道闪电是同时发生的。如果我问你这个说法是否有道理，你会肯定地回答我“是”。但是如果我请求你更准确地向我解释这句话的含义，你经过一番考虑，会发现这个问题的答案并非像乍看起来那么容易。

After some time perhaps the following answer would occur to you: "The significance of the statement is clear in itself and needs no further explanation; of course it would require some consideration if I were to be commissioned to determine by observations whether in the actual case the two events took place simultaneously or not." I cannot be satisfied with this answer for the following reason. Supposing that as a result of ingenious considerations an able meteorologist were to discover that the lightning must always strike the places A and B simultaneously, then we should be faced with the task of testing whether or not this theoretical result is in accordance with the reality. We encounter the same difficulty with all physical statements in which the conception " simultaneous " plays a part. The concept does not exist for the physicist until he has the possibility of discovering whether or not it is fulfilled in an actual case. We thus require a definition of simultaneity such that this definition supplies us with the method by means of which, in the present case, he can decide by experiment whether or not both the lightning strokes occurred simultaneously. As long as this requirement is not satisfied, I allow myself to be deceived as a physicist (and of course the same applies if I am not a physicist), when I imagine that I am able to attach a meaning to the statement of simultaneity. (I would ask the reader not to proceed farther until he is fully convinced on this point.)

一段时间后，你可能会想到以下答案：“这句话的意义本身已经很清楚，不需要再进一步的解释了。当然如果要我通过观察来确定两件事是否真正同时发生的话，还需要一些条件。”我对这个答案并不满意，原因如下。假设一个聪明的气象学家通过巧妙的解法，发现闪电总是同时击中两个地方 A 和 B，那么我们面临的任务就是检查这个理论结果是否符合现实。在所有与“同时性”这个概念有关的物理陈述中，我们都会遇到同样的困难。在有机会在实际中检验它之前，同时性这个概念对物理学家来说并不存在。因此，我们需要一个同时性的定义，这个定义能为我们提供一种方法，在当前的例子里，它可以帮我们用实验决定两个闪电是否同时发生。只要不满足这个要求，作为一名物理学家，我允许自己被欺骗（当然如果我不是物理学家也一样），想象自己能够给同时性赋予含义。（我会要求读者在完全确信这一点之前，不要再继续下去。）

After thinking the matter over for some time you then offer the following suggestion with which to test simultaneity. By measuring along the rails, the connecting line AB should be measured up and an observer placed at the mid-point M of the distance AB. This observer should be supplied with an arrangement (e.g. two mirrors inclined at 90^0) which allows him visually to observe both places A and B at the same time. If the observer perceives the two flashes of lightning at the same time, then they are simultaneous.

在考虑了一段时间之后，您提供了以下建议以用于测试同时性。通过沿着轨道测量，找到连线 AB 的中点 M，并让一个观察者站在该中点 M 上。该观察者有一个设备（例如两个成 90°的镜子），可以让他同时观察位置 A 和 B。如果观察者同时看到两束光，那么它们就是同时的。

I am very pleased with this suggestion, but for all that I cannot regard the matter as quite settled, because I feel constrained to raise the following objection:

我对这个建议很满意，但还不能认为这件事已经解决了，因为我觉得有必要提出以下反对意见：

"Your definition would certainly be right, if only I knew that the light by means of which the observer at M perceives the lightning flashes travels along the length A arrow M with the same velocity as along the length B arrow M. But an examination of this supposition would only be possible if we already had at our disposal the means of measuring time. It would thus appear as though we were moving here in a logical circle."

“要想保证你的定义是对的，就要求观察者用来感知的光束从 A 到 M 的速度与从 B 到 M 的速度必须相同。但是只有当我们已经知道怎样测量时间后，才能对这个假设进行检验。所以我们在这里好像掉进了一个逻辑循环的陷阱。”

After further consideration you cast a somewhat disdainful glance at me -- and rightly so -- and you declare:

经过再进一步的考虑之后，你有些不屑地看了我一眼，（这是理所当然的），你作出了如下声明：

"I maintain my previous definition nevertheless, because in reality it assumes absolutely nothing about light. There is only one demand to be made of the definition of simultaneity, namely, that in every real case it must supply us with an empirical decision as to whether or not the conception that has to be defined is fulfilled. That my definition satisfies this demand is indisputable. That light requires the same time to traverse the path A arrow M as for the path B arrow M is in reality neither a supposition nor a hypothesis about the physical nature of light, but a stipulation which I can make of my own freewill in order to arrive at a definition of simultaneity."

“尽管如此，我仍然保持我以前的定义，因为实际上它对光完全没有假设。对于同时性的定义，只有一个要求，它就是，针对每一个实际情况，它必须为我们提供一个经验决定，看需要定义的概念是否得到了满足。我的定义能满足这个需求是无可争辩的。光需要同样的时间通过路径 AM 和 BM，其实既不是关于光的本质的猜想也不是假设，而是我根据自己的自由意志做出的规定，以便实现对同时性的定义。”

It is clear that this definition can be used to give an exact meaning not only to two events, but to as many events as we care to choose, and independently of the positions of the scenes of the events with respect to the body of reference * (here the railway embankment). We are thus led also to a definition of " time " in physics. For this purpose we suppose that clocks of identical construction are placed at the points A, B and C of the railway line (co-ordinate system) and that they are set in such a manner that the positions of their pointers are simultaneously (in the above sense) the same. Under these conditions we understand by the " time " of an event the reading (position of the hands) of that one of these clocks which is in the immediate vicinity (in space) of the event. In this manner a time-value is associated with every event which is essentially capable of observation.

很明显，这个定义不但可以用来精确定义两个事件，而且可以用来定义任意多个事件，并且与事件相对于参考系的位置无关*（这里是指铁路路堤）。同时，我们还可以得出物理学中对“时间”的定义。为此，我们假设三个相同结构的时钟被放置在铁路线（坐标系）的 A、B 和 C 点上，并且它们的指针位置完全相同。具备了这些条件，我们就可以把事件的时间用它旁边的时钟的读数（指针的位置）来代表。通过这种这种方式，每个事件都被赋予一个时间值，并且这些时间值都是可以观测的。

This stipulation contains a further physical hypothesis, the validity of which will hardly be doubted without empirical evidence to the contrary. It has been assumed that all these clocks go at the same rate if they are of identical construction. Stated more exactly: When two clocks arranged at rest in different places of a reference-body are set in such a manner that a particular position of the pointers of the one clock is simultaneous (in the above sense) with the same position, of the pointers of the other clock, then identical " settings " are always simultaneous (in the sense of the above definition).

该规定包含了另一个物理假设，该假设的有效性几乎不会有人怀疑，除非发现了与之相反的经验证据。这个假设就是：构造完全相同的时钟都以同样的快慢运转。更准确地说：对于两个相对于参考系静止但处于不同位置的时钟，如果一个时钟指针的位置总是同时与另一个时钟的指针位置相同，那么完全相同的“设置”就总是同时的。

Notes

*) We suppose further, that, when three events A, B and C occur in different places in such a manner that A is simultaneous with B and B

is simultaneous with C (simultaneous in the sense of the above definition), then the criterion for the simultaneity of the pair of events A, C is also satisfied. This assumption is a physical hypothesis about the law of propagation of light: it must certainly be fulfilled if we are to maintain the law of the constancy of the velocity of light in vacuo.

注

*）我们进一步假设，对于发生在不同地方的三个事件 A、B 和 C，A 与 B 同时发生，B 与 C 也同时发生，（根据上述同时性的的定义），那么按照同时性的标准，事件 A、C 也是同时发生的。这个假定是一个在物理上对光的传播定律的假设，如果要保证在真空中光速的恒定，这个假定就一定成立。

THE RELATIVITY OF SIMULATNEITY

09. 同时性的相对性

Up to now our considerations have been referred to a particular body of reference, which we have styled a " railway embankment." We suppose a very long train travelling along the rails with the constant velocity v and in the direction indicated in Fig 1. People travelling in this train will with a vantage view the train as a rigid reference-body (co-ordinate system); they regard all events in

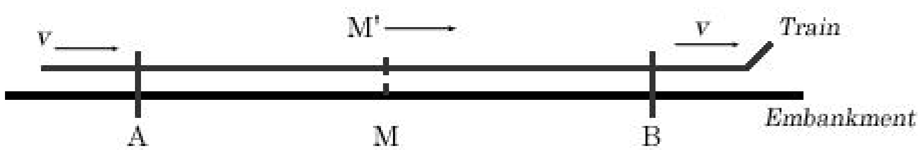

FIG. I.

reference to the train. Then every event which takes place along the line also takes place at a particular point of the train. Also the definition of simultaneity can be given relative to the train in exactly the same way as with respect to the embankment. As a natural consequence, however, the following question arises :

到目前为止，我们的讨论都是针对“铁路路堤”这个特定的参考系进行的。我们假设有一列很长的火车，正在沿着轨道以恒定的速度 v，按图示 1 所指示的方向行驶。这列火车上的人，可以方便地把这列火车当作参考系（也叫坐标系），他们讲的所有事件都是相对于该火车而言的。这样发生在沿线的每个事件也同样发生在火车上的某个特定点。所以我们也可以根据列车来定义同时性，就像根据路堤来定义时一模一样。这样做的结果，就引出了以下问题：

Are two events (e.g. the two strokes of lightning A and B) which are simultaneous with reference to the railway embankment also simultaneous relatively to the train? We shall show directly that the answer must be in the negative.

是否有两个事件（例如两次闪电 A 和 B），既相对于路堤是同时的，又相对于火车是同时的？我们下面将说明，对这个问题的答案一定是否定的。

When we say that the lightning strokes A and B are simultaneous with respect to be embankment, we mean: the rays of light emitted at the places A and B, where the lightning occurs, meet each other at the mid-point M of the length A arrow B of the embankment. But the events A and B also correspond to positions A and B on the train. Let M1 be the mid-point of the distance A arrow B on the travelling train. Just when the flashes (as judged from the embankment) of lightning occur, this point M1 naturally coincides with the point M but it moves towards the right in the diagram with the velocity v of the train. If an observer sitting in the position M1 in the train did not possess this velocity, then he would remain permanently at M, and the light rays emitted by the flashes of lightning A and B would reach him simultaneously, i.e. they would meet just where he is situated. Now in reality (considered with reference to the railway embankment) he is hastening towards the beam of light coming from B, whilst he is riding on ahead of the beam of light coming from A. Hence the observer will see the beam of light emitted from B earlier than he will see that emitted from A. Observers who take the railway train as their reference-body must therefore come to the conclusion that the lightning flash B took place earlier than the lightning flash A. We thus arrive at the important result:

当我们说闪电 A 和 B 相对于路堤同时发生时，我们的意思是：当闪电 A 和 B 发生时，从它们发出的光线正好同时到达路堤上 A 和 B 的中点 M。但事件 A 和 B 也对应于列车上的位置 A 和 B。假设 M′ 是行进中的列车上 A 和 B 的中点。当闪电（从路堤判断）发生时，这个点 M′ 点 M 重合，

不过它在以火车的速度 v 向图中的右侧移动。如果坐在火车上 M′ 位置的观察者没有这个速度，那么他将永远呆在 M 处，闪电 A 和 B 发出的光线就会同时到达他所在的位置。事实上（以路堤作为参考），他是在朝着从 B 发出的光束快速移动，同时也在远离来自 A 的光束。因此观察者会早些看到从 B 发出的光束，而会晚些看到从 A 发出的光束。所以，以铁路列车做参考的观察者一定会得出这样的结论，即闪电 B 发生的时间早于闪电 A。由此，我们可以得出如下重要结论：

Events which are simultaneous with reference to the embankment are not simultaneous with respect to the train, and vice versa (relativity of simultaneity). Every reference-body (co-ordinate system) has its own particular time ; unless we are told the reference-body to which the statement of time refers, there is no meaning in a statement of the time of an event.

相对于路堤同时发生的事件相对于火车不是同时的，反之亦然（同时性的相对性）。每个参考系（坐标系）都有它自己特定的时间。除非我们知道时间是相对于哪个参考系的，否则讨论事件发生的时间就没有任何意义。

Now before the advent of the theory of relativity it had always tacitly been assumed in physics that the statement of time had an absolute significance, i.e. that it is independent of the state of motion of the body of reference. But we have just seen that this assumption is incompatible with the most natural definition of simultaneity; if we discard this assumption, then the conflict between the law of the propagation of light in vacuo and the principle of relativity (developed in Section 7) disappears.

在相对论出现之前，在物理学中我们总是默认时间具有绝对的意义，也即时间独立于参考系的运动状态。但我们刚刚看到，这个假设与同时性的最自然的定义不相容。如果我们放弃这个假设，那么光在真空中的传播规律和相对论原理的冲突（在第 7 节中论述的）就消失了。

We were led to that conflict by the considerations of Section 6, which are now no longer tenable. In that section we concluded that the man in the carriage, who traverses the distance w per second relative to the carriage, traverses the same distance also with respect to the embankment in each second of time. But, according to the foregoing considerations, the time required by a particular occurrence with respect to the carriage must not be considered equal to the duration of the same occurrence as judged from the embankment (as reference-body). Hence it cannot be contended that the man in walking travels the distance w relative to the railway line in a time which is equal to one second as judged from the embankment.

这种冲突是由我们在第 6 节的观点造成的，那些观点现在已经站不住脚了。在那一节里，我们认为如果车厢里的人以 w/s 的速度相对于车厢移动，那么它相对于路堤每秒也移动这么远的距离。但是，根据上述考虑，一个事件相对于车厢所需的时间不应该被认为等于它以路堤做参考系时所需的时间。因此，不能说相对于路堤来说，前面例子中的人在一秒内走了 w 的距离。

Moreover, the considerations of Section 6 are based on yet a second assumption, which, in the light of a strict consideration, appears to be arbitrary, although it was always tacitly made even before the introduction of the theory of relativity.

另外，第 6 节的讨论是基于另一个假定，严格来说它是随意选取的，不过在有相对论之前，我们对它是一直默认的。

ON THE RELATIVITY OF THE CONCEPTION OF DISTANCE

10. 关于距离概念的相对性

Let us consider two particular points on the train * travelling along the embankment with the velocity v, and inquire as to their distance apart. We already know that it is necessary to have a body of reference for the measurement of a distance, with respect to which body the distance can be measured up. It is the simplest plan to use the train itself as reference-body (co-ordinate system). An observer in the train measures the interval by marking off his measuring-rod in a straight line (e.g. along the floor of the carriage) as many times as is necessary to take him from the one marked point to the other. Then the number which tells us how often the rod has to be laid down is the required distance.

设想在以速度 v 沿着路堤行驶的火车*上有两个特殊的点，我们需要知道它们之间的距离。我们知道要测量距离，就必须要有一个参考系，距离就是按照它测量出来的。最简单的办法是使用列车本身作为参考系（坐标系）。一个在火车上的观察者用他的测量杠沿着直线（也即沿着车厢的地板）一杆接一杆地测量，直到把两点之间的间隔全量完。上面的测量所需要的次数就是我们想知道的距离。

It is a different matter when the distance has to be judged from the railway line. Here the following method suggests itself. If we call A^1 and B^1 the two points on the train whose distance apart is required, then both of these points are moving with the velocity v along the embankment. In the first place we require to determine the points A and B of the embankment which are just being passed by the two points A^1 and B^1 at a particular time t -- judged from the embankment. These points A and B of the embankment can be determined by applying the definition of time given in Section 8. The distance between these points A and B is then measured by repeated application of thee measuring-rod along the embankment.

当需要根据铁路线来判断该距离时，情况就不同了。我们可以考虑使用以下方法。如果我们把火车上需要测距的两点称为 A^1 和 B^1，那么这两个点都在以速度 v 沿着路堤移动。首先我们需要确定在某一时刻 t（火车上的）两点 A^1 和 B^1 所经过的路堤上的 A 点和 B 点（以路堤作参考）。这里的点 A 和点 B 可以用第 8 节中给出的时间定义来确定。然后通过用测量杆重复测量若干次，就能得到点 A 和 B 之间的距离。

A priori it is by no means certain that this last measurement will supply us with the same result as the first. Thus the length of the train as measured from the embankment may be different from that obtained by measuring in the train itself. This circumstance leads us to a second objection which must be raised against the apparently obvious consideration of Section 6. Namely, if the man in the carriage covers the distance w in a unit of time -- measured from the train, -- then this distance -- as measured from the embankment -- is not necessarily also equal to w.

先验告诉我们，后一次测量的结果不一定与第一次相同。因此沿路堤量出的长度可能与从火车上量出的长度不同。基于这种情形，我们必须对第 6 节的明显想法提出第二个反对意见。也就是说，如果一个车厢内的人在单位时间内移动了距离 w（以火车为准），那么沿着路堤测量，这个距离不一定也等于 w。

Notes

*) e.g. the middle of the first and of the hundredth carriage.

注：

*）比如说在第一节车厢和第一百节车厢的中间。

THE LORENTZ TRANSFORMATION

11. 洛伦兹变换

The results of the last three sections show that the apparent incompatibility of the law of propagation of light with the principle of relativity (Section 7) has been derived by means of a consideration which borrowed two unjustifiable hypotheses from classical mechanics; these are as follows:

前面三节的结果表明，光的传播定律与相对性原理的明显不相容（第 7 节），是因为借用了经典力学的两个不合理的假设而导致的。下面是这两个假设：

(1) The time-interval (time) between two events is independent of the condition of motion of the body of reference.

(1) 两个事件之间的时间间隔（这里称之为时间），与参照系的运动状态无关。

(2) The space-interval (distance) between two points of a rigid body is independent of the condition of motion of the body of reference.

(2) 一个刚体两点间的空间间隔（这里称之为距离），与参照系的运动状态无关。

If we drop these hypotheses, then the dilemma of Section 7 disappears, because the theorem of the addition of velocities derived in Section 6 becomes invalid. The possibility presents itself that the law of the propagation of light in vacuo may be compatible with the principle of relativity, and the question arises: How have we to modify the considerations of Section 6 in order to remove the apparent disagreement between these two fundamental results of experience? This question leads to a general one. In the discussion of Section 6 we have to do with places and times relative both to the train and to the embankment. How are we to find the place and time of an event in relation to the train, when we know the place and time of the event with respect to the railway embankment ? Is there a thinkable answer to this question of such a nature that the law of transmission of light in vacuo does not contradict the principle of relativity ? In other words : Can we conceive of a relation between place and time of the individual events relative to both reference-bodies, such that every ray of light possesses the velocity of transmission c relative to the embankment and relative to the train ? This question leads to a quite definite positive answer, and to a perfectly definite transformation law for the space-time magnitudes of an event when changing over from one body of reference to another.

如果我们放弃这些假设，那么第 7 节的难题就消失了，因为第 6 节中推导出的速度相加定理就失效了。这就有可能让光在真空中的传播定律与相对性原理相兼容，问题是：我们该如何修改第 6 节的情况，才能消除这两个基本经验结果之间的明显分歧？这个问题引出了一个普遍问题。在第 6 节的讨论中，我们得处理相对于火车和路堤两者的地点和时间问题。当我们已知一个事件相对于铁路路堤发生的地点和时间的时候，我们该如何找到该事件相对于火车发生的时间和地点？是否存在一种答案，使得光在真空中的传输规律与相对性原理不冲突？换句话说：我们能否想出一种在两个参考系之间的空间和时间关系，使得光线相对与路堤与火车都具有同样的传播速度 c？这个问题有一个非常肯定的答案：当把一个事件从一个参照系转换到另一个参照系时，存在一个完全确定的变换规律，用来变换事件在两个参照系中的时空坐标。

Before we deal with this, we shall introduce the following incidental consideration. Up to the present we have only considered events taking place along the embankment, which had mathematically to assume the function of a straight line. In the manner indicated in Section 2 we can imagine this reference-body supplemented laterally and in a vertical direction by means of a framework of rods, so that an event which takes place anywhere can be localised with reference to this framework.

在我们着手处理之前，我们先介绍下面的附带情况。到目前为止，我们只考虑了事件沿着路堤发生的情况，它在数学上是一个代表直线的函数。像第 2 节那样，我们可以想象这个参考体在横向和竖直方向都通过杆框架进行了扩展，这样任何地方发生的事件都可以用该系统进行定位。

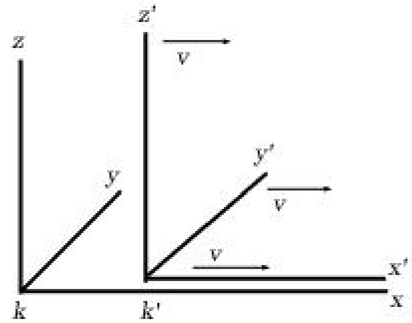

Fig. 2

Similarly, we can imagine the train travelling with the velocity v to be continued across the whole of space, so that every event, no matter how far off it may be, could also be localised with respect to the second framework. Without committing any fundamental error, we can disregard the fact that in reality these frameworks would continually interfere with each other, owing to the impenetrability of solid bodies. In every such framework we imagine three surfaces perpendicular to each other marked out, and designated as " co-ordinate planes " (" co-ordinate system "). A co-ordinate system K then corresponds to the embankment, and a co-ordinate system K' to the train. An event, wherever it may have taken place, would be fixed in space with respect to K by the three perpendiculars x, y, z on the co-ordinate planes, and with regard to time by a time value t. Relative to K1, the same event would be fixed in respect of space and time by corresponding values x1, y1, z1, t1, which of course are not identical with x, y, z, t. It has already been set forth in detail how these magnitudes are to be regarded as results of physical measurements.

同样，我们可以想象以速度 v 运行的火车跨越了整个空间，这样每个事件，无论它距离有多远，都可以在第二个框架里进行定位。在现实中，由于固体的不可穿透性，这些框架会不断地相互干扰。不过我们可以完全忽略这种情况，而不引进任何根本性错误。想象在这样的框架中，我们标出了三个相互垂直的表面，并称之为“坐标平面”（“坐标系”）。一个坐标系 K 对应于路堤，另一个坐标系 K′ 对应于火车。一个事件，无论它在哪里发生，都可用三个坐标 x, y, z 来确定其相对于 K 的空间位置，并用时间值 t 来确定其时间。相对于 K′，同一事件的空间和时间可以用相应的 x′, y′, z′, t′ 来确定，当然它们与 x, y, z, t 不同。关于这些值的物理测量方法，前面已经已经详细解释过了。

Obviously our problem can be exactly formulated in the following manner. What are the values x1, y1, z1, t1, of an event with respect to K1, when the magnitudes x, y, z, t, of the same event with respect to K are given ? The relations must be so chosen that the law of the transmission of light in vacuo is satisfied for one and the same ray of light (and of course for every ray) with respect to K and K1. For the relative orientation in space of the co-ordinate systems indicated in the diagram ([7]Fig. 2), this problem is solved by means of the equations :

显然，我们的问题可以精确地表述为以下形式：一个事件相对于 K1 的值 x1、y1、z1、t1 是什么，如果已知同一事件相对于 K 的 x, y, z, t 的值？选择这个关系时，我们必须保证同一条光线必须相对于 K 和 K1 都满足光在真空中的传播规律。对于（[7]图 2）所示的坐标系之间的相对关系，这个问题可以用下述方程解决：

$$x' = \frac{x - vt}{\sqrt{I - \frac{v^2}{c^2}}}$$

y1 = y
z1 = z

$$t' = \frac{t - \frac{v}{c^2} \cdot x}{\sqrt{I - \frac{v^2}{c^2}}}$$

This system of equations is known as the " Lorentz transformation." *

If in place of the law of transmission of light we had taken as our basis the tacit assumptions of the older mechanics as to the absolute character of times and lengths, then instead of the above we should have obtained the following equations:

这个方程组被称为“洛伦兹变换”。*

如果我们不用光的传播定律，而是以旧力学中默认的绝对时间和长度为基础，那么我们得到的就不是上面的结果，而是下述公式：

x1 = x - vt
y1 = y
z1 = z

$$t1 = t$$

This system of equations is often termed the " Galilei transformation." The Galilei transformation can be obtained from the Lorentz transformation by substituting an infinitely large value for the velocity of light c in the latter transformation.

这个方程组通常被称为“伽利略变换”。如果把洛伦兹变换中的光速 c 替换为无穷大，就可以得到伽利略变换 。

Aided by the following illustration, we can readily see that, in accordance with the Lorentz transformation, the law of the transmission of light in vacuo is satisfied both for the reference-body K and for the reference-body K1. A light-signal is sent along the positive x-axis, and this light-stimulus advances in accordance with the equation

借助下图，我们可以立马看出，按照洛伦兹变换，真空中的光的传播规律相对于参考物 K 和 K1 都能得到满足。沿着正 x 轴的方向发出一个光信号，它所引起的干扰按照等式

$$x = ct,$$

i.e. with the velocity c. According to the equations of the Lorentz transformation, this simple relation between x and t involves a relation between x1 and t1. In point of fact, if we substitute for x the value ct in the first and fourth equations of the Lorentz transformation, we obtain:

进行传播，也即以速度 c 进行传播。根据洛伦兹变换的公式，x 和 t 之间的这种简单关系与 x1 和 t1 之间的关系有关。事实上，如果我们把洛伦兹变换的第一和第四个方程中的值 x 替换为 ct，就可以得到：

$$x' = \frac{(c - v)t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t' = \frac{\left(1 - \frac{v}{c}\right)t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

from which, by division, the expression

把以上两式相除，马上就可以得到下式：

$$x1 = ct1$$

immediately follows. If referred to the system K1, the propagation of light takes place according to this equation. We thus see that the velocity of transmission relative to the reference-body K1 is also equal to c. The same result is obtained for rays of light advancing in any other direction whatsoever. Of cause this is not surprising, since the equations of the Lorentz transformation were derived conformably to this point of view.

相对于参考系 K1，光就按照这个公式进行传播。因此我们可以看出，相对于参考系 K1，光的传输速度也等于 c。对于任何沿其它方向前进的光， 我们也可以得相同的结果。当然这并不奇怪，因为洛伦兹变换的方程就是按此推导出来的。

Notes

*) A simple derivation of the Lorentz transformation is given in Appendix I.

注

*）附录一提供了对洛伦兹变换的简单推导。

THE BEHAVIOUR OF MEASURING-RODS AND CLOCKS IN MOTION

12. 测量杆和时钟在运动中的表现

Place a metre-rod in the x1-axis of K1 in such a manner that one end (the beginning) coincides with the point x1=0 whilst the other end (the end of the rod) coincides with the point x1=I. What is the length of the metre-rod relatively to the system K? In order to learn this, we need only ask where the beginning of the rod and the end of the rod lie with respect to K at a particular time t of the system K. By means of the first equation of the Lorentz transformation the values of these two points at the time t = 0 can be shown to be

在 K1 的 x1 轴上放置一根米尺，使其一端（起始端）与点 x1=0 重合，而另一端（杆的末端）与点 x1=1 重合。相对于系统 K，该米尺的长度是多少？为了回答这个问题，我们只需要知道在某一特定时间 t，杆的起点和终点相对于 K 处于什么位置。根据洛伦兹变换的第一个方程，这两点在时刻 t=0 的值可以表示为

$$x_{\text{(begining of rod)}} = 0 \sqrt{I - \frac{v^2}{c^2}}$$

$$x_{\text{(end of rod)}} = I \sqrt{I - \frac{v^2}{c^2}}$$

the distance between the points being $\sqrt{I - v^2/c^2}$

两点之间的距离是$\sqrt{I - v^2/c^2}$

But the metre-rod is moving with the velocity v relative to K. It therefore follows that the length of a rigid metre-rod moving in the direction of its length with a velocity v is $\sqrt{I - v^2/c^2}$ of a metre.

但是米尺正以速度 v 相对于 K 在运动，所以一个以速度 v 沿其长度方向运动的刚性米尺的长度是$\sqrt{I - v^2/c^2}$米。

The rigid rod is thus shorter when in motion than when at rest, and the more quickly it is moving, the shorter is the rod. For the velocity v=c we should have $\sqrt{I - v^2/c^2} = 0$,

因此，刚性杆在运动时比静止时短，并且移动得越快，杆就越短。当速度 v=c 时，我们就得到$\sqrt{I - v^2/c^2} = 0$

and for stiII greater velocities the square-root becomes imaginary. From this we conclude that in the theory of relativity the velocity c plays the part of a limiting velocity, which can neither be reached nor exceeded by any real body.

对于更大的速度，平方根就变成虚数了。由此我们得出结论，在相对论中，速度 c 扮演着极限速度的角色，任何物体都达不到更超不过该速度。

Of course this feature of the velocity c as a limiting velocity also clearly follows from the equations of the Lorentz transformation, for these became meaningless if we choose values of v greater than c.

当然，速度 c 作为极限速度的这个作用，也可以从洛伦兹变换的方程里清楚地看出。因为如果我们选取大于 c 的 v 值，它们就没有意义了。

If, on the contrary, we had considered a metre-rod at rest in the x-axis with respect to K, then we should have found that the length of

the rod as judged from K1 would have been $\sqrt{I - v^2/c^2}$;
如果反过来，我们假设米尺相对于 K 的 x 轴静止，那么它相对于 K1 的长度就变成了$\sqrt{I - v^2/c^2}$。

this is quite in accordance with the principle of relativity which forms the basis of our considerations.
这完全符合相对性原理，而相对性原理是我们讨论的基础。

A Priori it is quite clear that we must be able to learn something about the physical behaviour of measuring-rods and clocks from the equations of transformation, for the magnitudes z, y, x, t, are nothing more nor less than the results of measurements obtainable by means of measuring-rods and clocks. If we had based our considerations on the Galileian transformation we should not have obtained a contraction of the rod as a consequence of its motion.
很明显，从这些变换公式，我们可以看出测量杆和时钟的物理表现，因为由此得出的 z, y, x, t 的大小与通过测量杆和时钟实际测得的值丝毫不差。如果我们利用伽利略变换，就不会得到杆因为运动而收缩的结论。

Let us now consider a seconds-clock which is permanently situated at the origin (x1=0) of K1. t1=0 and t1=I are two successive ticks of this clock. The first and fourth equations of the Lorentz transformation give for these two ticks :
现在假设有一个永远呆在 K1 原点(x1=0)的秒表。t1=0 和 t1=1 是该表上两个连续的刻度。洛伦兹变换的第一和第四方程给出了这两个刻度的值：

$$t = 0$$

And
和

$$t' = \frac{I}{\sqrt{I - \frac{v^2}{c^2}}}$$

As judged from K, the clock is moving with the velocity v; as judged from this reference-body, the time which elapses between two strokes of the clock is not one second, but

从 K 来看，时钟正在以速度 v 移动。所以在这个参照系里，钟表两次滴答之间经过的时间不是一秒，而是

$$\frac{I}{\sqrt{I-\frac{v^2}{c^2}}}$$

seconds, i.e. a somewhat larger time. As a consequence of its motion the clock goes more slowly than when at rest. Here also the velocity c plays the part of an unattainable limiting velocity.

秒，也即稍大点儿的时间。由于运动的缘故，时钟比它不动时走得要慢。这里的速度 c 也扮演着无法达到的极限速度的角色。

THEOREM OF THE ADDITION OF VELOCITIES. THE EXPERIMENT OF FIZEAU

13. 速度相加定理和菲索实验

Now in practice we can move clocks and measuring-rods only with velocities that are small compared with the velocity of light; hence we shall hardly be able to compare the results of the previous section directly with the reality. But, on the other hand, these results must strike you as being very singular, and for that reason I shall now draw another conclusion from the theory, one which can easily be derived from the foregoing considerations, and which has been most elegantly confirmed by experiment.

在现实中，我们只能用与光速相比很小的速度来移动时钟和测量杆，所以我们几乎无法把上一节的结果与现实相比较。但是，从另一方面讲，这些结果肯定让你感觉非常独别，因此我现在要提出该理论的另一个结论，该结论可以从前面的条件很容易地推导出来，并且已经被实验漂亮地证实了。

In Section 6 we derived the theorem of the addition of velocities in one direction in the form which also results from the hypotheses of classical mechanics- This theorem can also be deduced readily horn the Galilei transformation (Section 11). In place of the man walking inside the carriage, we introduce a point moving relatively to the co-ordinate system K1 in accordance with the equation

在第 6 节中，我们推导出了单一方向上的速度相加定理，它同时也是经典力学的假设的结果。这个定理也可以直接从伽利略变换（第 11 节）推导出来。这次我们不用在车厢里行走的人，而是用一个相对于坐标系 K1 移动的点，下面是它的移动公式：

$$x1 = wt1$$

By means of the first and fourth equations of the Galilei transformation we can express x1 and t1 in terms of x and t, and we then obtain

利用伽利略变换的第一和第四方程，我们可以用 x 和 t 来表示 x1 和 t1，于是得到

$$x = (v + w)t$$

This equation expresses nothing else than the law of motion of the point with reference to the system K (of the man with reference to the embankment). We denote this velocity by the symbol W, and we then obtain, as in Section 6,

这个方程代表了该点相对于参考系统 K（人相对于路堤）的运动规律。用符号 W 来表示这个速度，于是就像第 6 节那样，我们得到了如下公式：

$$W=v+w \qquad A)$$

$$W=v+w \qquad A)$$

But we can carry out this consideration just as well on the basis of the theory of relativity. In the equation

但是我们也可以基于相对论原理进行展开。对于公式

$$x1=wt1 \qquad B)$$

$$x1 = wt1 \qquad B)$$

we must then express x1and t1 in terms of x and t, making use of the first and fourth equations of the Lorentz transformation. Instead of the equation (A) we then obtain the equation

我们要利用洛伦兹变换的第一和第四方程，用 x 和 t 来表示 x1 和 t1。于是我们得到的不是公式(A)，而是公式

$$W = \frac{v + w}{I + \frac{vw}{c^2}}$$

which corresponds to the theorem of addition for velocities in one direction according to the theory of relativity. The question now arises as to which of these two theorems is the better in accord with experience. On this point we axe enlightened by a most important experiment which the brilliant physicist Fizeau performed more than half a century ago, and which has been repeated since then by some of the best experimental physicists, so that there can be no doubt about its result. The experiment is concerned with the following question. Light travels in a motionless liquid with a particular velocity w. How quickly does it travel in the direction of the arrow in the tube T

这就是相对论下单一方向上的速度相加定理。现在的问题是，这两个定理哪个更符合实验结果。在这点上，我们可以从

出现这两个定理中哪个更符合经验。在这一点上，我们可以从杰出物理学家菲索的重要实验得到启发。该实验是半个世纪以前做的，之后一些最好的实验物理学家又重复了这个实验，所以该实验的结果是毋庸置疑的。该实验研究的是以下问题。光在静止的液体中以特定的速度 w 传播。如果管道中的液体以速度 v 流动，光在管道中的传播速度是多少？

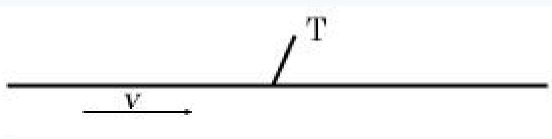

Fig. 3

when the liquid above mentioned is flowing through the tube with a velocity v ?

In accordance with the principle of relativity we shall certainly have to take for granted that the propagation of light always takes place with the same velocity w with respect to the liquid, whether the latter is in motion with reference to other bodies or not. The velocity of light relative to the liquid and the velocity of the latter relative to the tube are thus known, and we require the velocity of light relative to the tube.

根据相对性原理，不管流体是否相对于参照系在运动，光相对于流体的传播速度总是同样的速度 w。所以光相对于流体的速度和流体相对于管子的速度都是已知的，我们这里要求的是光相对于管子的速度。

It is clear that we have the problem of Section 6 again before us. The tube plays the part of the railway embankment or of the co-ordinate system K, the liquid plays the part of the carriage or of the co-ordinate system K1, and finally, the light plays the part of the man walking along the carriage, or of the moving point in the present section. If we denote the velocity of the light relative to the tube by W, then this is given by the equation (A) or (B), according as the Galilei transformation or the Lorentz transformation corresponds to the facts. Experiment * decides in favour of equation (B) derived from the theory of relativity, and the agreement is, indeed, very exact. According to recent and most excellent measurements by Zeeman, the influence of the velocity of flow v on the propagation of light is represented by formula (B) to within one per cent.

很明显，我们又再次面临着第 6 节的问题。管子扮演着铁路路堤或坐标系统 K 的角色，液体扮演着车厢或坐标系 K1 的角色，最后，光扮演着车厢里行走的人，或对本节来说，它代表着移动的那个点。如果我们用 W 表示光相对于管子的速度，那么这个速度可由公式（A）或（B）给出，一个是伽利略变换，另一个是洛伦兹变换。实验结果*支持相对论导出的公式(B)，并且非常精确。根据塞曼最近和最好的测量结果，流速 v 对光传播的影响与由公式(B)得出的值，误差在百分之一以内。

Nevertheless we must now draw attention to the fact that a theory of this phenomenon was given by H. A. Lorentz long before the statement of the theory of relativity. This theory was of a purely electrodynamical nature, and was obtained by the use of particular hypotheses as to the electromagnetic structure of matter. This circumstance, however, does not in the least diminish the conclusiveness of the experiment as a crucial test in favour of the theory of relativity, for the electrodynamics of Maxwell-Lorentz, on which the original theory was based, in no way opposes the theory of relativity. Rather has the latter been developed trom electrodynamics as an astoundingly simple combination and generalisation of the hypotheses, formerly independent of each other, on which electrodynamics was built.

不过，现在必须提请大家注意一个事实，早在相对论之前，H. A.洛伦兹就对此现象给出了一个解释。这个解释是纯粹的电动力学性质的，它是通过关于物质电磁结构的特殊假设得到的。然而这个解释丝毫并不减少该实验结果对相对论的支持，因为该解释基于麦克斯韦-洛伦兹的电动力学，它绝不反对相对论。相反，相对论对这些相互独立的假设做了异常简单的组合和概括，而电动力学就是建立在这些假设之上的。

Notes

*) Fizeau found ,

$$W = w + v\left(I - \frac{I}{n^2}\right)$$

where $n = \frac{c}{w}$

is the index of refraction of the liquid. On the other hand, owing to the smallness of $\frac{vw}{c^2}$ as compared with I,

we can replace (B) in the first place by $W = (w + v)\left(I - \frac{vw}{c^2}\right)$, or to the same order of approximation by

$w + v\left(I - \frac{I}{n^2}\right)$, which agrees with Fizeau's result.

注：

*）斐索发现：$W = w + v\left(I - \frac{I}{n^2}\right)$

这里的 $n = \frac{c}{w}$ 是流体的折射率。

另一方面，由于 $\frac{vw}{c^2}$ 与 1 相比小很多，我们可以将(B)

替换为 $W = (w + v)\left(I - \frac{vw}{c^2}\right)$ ，

它与 $w + v\left(I - \frac{I}{n^2}\right)$ 是在同一级别上的近似，

也就是与菲索的结果一致。

THE HEURISTIC VALUE OF THE THEORY OF RELATIVITY

14. 相对论的启发价值

Our train of thought in the foregoing pages can be epitomised in the following manner. Experience has led to the conviction that, on the one hand, the principle of relativity holds true and that on the other hand the velocity of transmission of light in vacuo has to be considered equal to a constant c. By uniting these two postulates we obtained the law of transformation for the rectangular co-ordinates x, y, z and the time t of the events which constitute the processes of nature. In this connection we did not obtain the Galilei transformation, but, differing from classical mechanics, the Lorentz transformation.

我们在前面几页中的思路可以用以下方式进行概括。经验使我们确信，一方面，相对性原理是成立的，另一方面，光在真空中的传播速度必等于一个常数 c。把这两个假设合起来，对于自然界发生的事件, 我们就得到了对其直角坐标 x、y、z 和时间 t 进行变换的规律。从这种结合，我们得到的不是伽利略变换，而是与经典力学不同的洛伦兹变换。

The law of transmission of light, the acceptance of which is justified by our actual knowledge, played an important part in this process of thought. Once in possession of the Lorentz transformation, however, we can combine this with the principle of relativity, and sum up the theory thus:

光的传播定律，（基于我们的实际知识我们接受了它），在这个思考过程中发挥了重要作用。一旦拥有了洛伦兹变换，我们就可以把它与相对性原理相结合，并总结出如下结论：

Every general law of nature must be so constituted that it is transformed into a law of exactly the same form when, instead of the space-time variables x, y, z, t of the original coordinate system K, we introduce new space-time variables x1, y1, z1, t1 of a co-ordinate system K1. In this connection the relation between the ordinary and the accented magnitudes is given by the Lorentz transformation. Or in brief : General laws of nature are co-variant with respect to Lorentz transformations.

每一个自然定律都必须表达为如下形式：当用坐标系 K1 中的时空变量 x1， y1， z1， t1 代替坐标系 K 中的时空变量 x， y， z， t 时, 自然定律的形式应该保持不变。在这方面，这两个坐标值之间的关系由洛伦兹变换给出。简单来说就是：一般自然定律相对于洛伦兹变换是协变的。

This is a definite mathematical condition that the theory of relativity demands of a natural law, and in virtue of this, the theory becomes a valuable heuristic aid in the search for general laws of nature. If a general law of nature were to be found which did not satisfy this condition, then at least one of the two fundamental assumptions of the theory would have been disproved. Let us now examine what general results the latter theory has hitherto evinced.

相对论要求自然法则必须满足这个确定的数学条件，也正因为如此，相对论在寻找普遍的自然规律的时候可起到宝贵的启发式的帮助作用。如果能找到不满足这个条件的一个普遍的自然法则，那么这两个基本假设中至少有一个会被推翻。我们现在来看看本理论迄今已经展示的普遍结果。

GENERAL RESULTS OF THE THEORY

15. 该理论的普遍结果

It is clear from our previous considerations that the (special) theory of relativity has grown out of electrodynamics and optics. In these fields it has not appreciably altered the predictions of theory, but it has considerably simplified the theoretical structure, i.e. the derivation of laws, and -- what is incomparably more important -- it has considerably reduced the number of independent hypothese forming the basis of theory. The special theory of relativity has rendered the Maxwell-Lorentz theory so plausible, that the latter would have been generally accepted by physicists even if experiment had decided less unequivocally in its favour.

从我们之前的讨论可以清楚地看出，（狭义）相对论是起源于电动力学和光学的。在这些领域，它并没有明显改变理论的预测，但是却大大简化了理论结构，也即物理定律的推导，以及，更为重要的是，它大大减少了作为理论基础的独立假设的数量。狭义相对论把麦克斯韦-洛伦兹理论变得如此可信，以至于物理学家已经普遍接受了它，即便实验没有那么明确地支持它。

Classical mechanics required to be modified before it could come into line with the demands of the special theory of relativity. For the main part, however, this modification affects only the laws for rapid motions, in which the velocities of matter v are not very small as compared with the velocity of light. We have experience of such rapid motions only in the case of electrons and ions; for other motions the variations from the laws of classical mechanics are too small to make themselves evident in practice. We shall not consider the motion of stars until we come to speak of the general theory of relativity. In accordance with the theory of relativity the kinetic energy of a material point of mass m is no longer given by the well-known expression

经典力学需要经过修改才能满足狭义相对论的要求。不过，这种修改只影响高速运动，也就是物体的运动速度 v 相对于光速来说不是很小的时候。只有在处理电子和离子的时候，我们才能遇到这种高速运动。对于其它（低速）运动，它与经典力学定律的差别太小，在实践中显示不出来。在讨论广义相对论之前，我们暂不考虑恒星的运动。按照相对论，一个质量为 m 的质点的动能不再是众所周知的

$$m\frac{v^2}{2}$$

$$m\frac{v^2}{2}$$

but by the expression

而是

$$\frac{mc^2}{\sqrt{I-\frac{v^2}{c^2}}}$$

$$\frac{mc^2}{\sqrt{I-\frac{v^2}{c^2}}}$$

This expression approaches infinity as the velocity v approaches the velocity of light c. The velocity must therefore always remain less than c, however great may be the energies used to produce the acceleration. If we develop the expression for the kinetic energy in the form of a series, we obtain

当速度 v 接近光速 c 时，该式的值趋于无穷。因此，不管用于产生加速度的能量有多大, 该速度总是始终小于 c。如果我们把动能表达式表达为序列的形式，就可以得到

$$mc^2+m\frac{v^2}{2}+\frac{3}{8}m\frac{v^4}{c^2}+\ldots$$

$$mc^2+m\frac{v^2}{2}+\frac{3}{8}m\frac{v^4}{c^2}+\ldots$$

When $\frac{v^2}{c^2}$ is small compared with unity, the third of these terms is always small in comparison with the second,

当 $\frac{v^2}{c^2}$ 与 1 相比较小时，这些项中的第三项总比第二项小，

which last is alone considered in classical mechanics. The first term mc^2 does not contain the velocity, and requires no consideration if we are only dealing with the question as to how the energy of a point-mass; depends on the velocity. We shall speak of its essential significance later.

这第二项是经典力学中唯一考虑的。第一项 mc^2 不包含速度 v，如果我们只考虑质点能量与速度的关系，可以把它忽略掉。后面我们会讲到它的重要性。

The most important result of a general character to which the special theory of relativity has led is concerned with the conception of mass. Before the advent of relativity, physics recognised two conservation laws of fundamental importance, namely, the law of the canservation of energy and the law of the conservation of mass these two fundamental laws appeared to be quite independent of each other. By means of the theory of relativity they have been united into one law. We shall now briefly consider how this unification came about, and what meaning is to be attached to it.

狭义相对论的一个最重要的普适性的结果与质量的概念有关。在相对论出现之前，物理学里有两个重要的定律，也即能量守恒定律和质量守恒定律。这两个基本定律似乎是彼此相互独立的。通过相对论，它们已经合为了一个定律。我们下面简单地讨论一下这种统一是如何产生的，以及这种统一的意义是什么。

The principle of relativity requires that the law of the concervation of energy should hold not only with reference to a co-ordinate system K, but also with respect to every co-ordinate system K1 which is in a state of uniform motion of translation relative to K, or, briefly, relative to every " Galileian " system of co-ordinates. In contrast to classical mechanics; the Lorentz transformation is the deciding factor in the transition from one such system to another.

相对性原理要求能量守恒定律不仅在坐标系 K 里成立，而且对于每一个相对于 K 做匀速直线运动的坐标系 K1，或简称为“伽利略”坐标系，都得成立。与经典力学相比，洛伦兹变换是从一个系统过渡到另一个系统的决定性因素。

By means of comparatively simple considerations we are led to draw the following conclusion from these premises, in conjunction with the fundamental equations of the electrodynamics of Maxwell: A body moving with the velocity v, which absorbs * an amount of energy E[0] in the form of radiation without suffering an alteration in velocity in the process, has, as a consequence, its energy increased by an amount

通过比较简单的考虑，我们从这些前提，再加上麦克斯韦电动力学的基本方程，可以得出如下结论：如果一个以速度 v 运动的物体，以辐射的形式吸收*了能量 E[0]，但其速度并未变化，那么在该过程中，它的能量增加了以下这么多

$$\frac{E_0}{\sqrt{I-\frac{v^2}{c^2}}}$$

In consideration of the expression given above for the kinetic energy of the body, the required energy of the body comes out to be

考虑到上面给出的动能表达式，物体的总体能量就是

$$\frac{\left(m+\frac{E_0}{c^2}\right)c^2}{\sqrt{I-\frac{v^2}{c^2}}}$$

Thus the body has the same energy as a body of mass

因此，该物体的能量，与以速度 v 移动并具有如下质量的物体所拥有的能量相同

$$\left(m+\frac{E_0}{c^2}\right)$$

moving with the velocity v. Hence we can say: If a body takes up an amount of energy E[0], then its inertial mass increases by an amount

因此我们可以说：如果一个物体接收了能量 E[0]，那么它的惯性质量就增加了

$$\frac{E_0}{c^2}$$

the inertial mass of a body is not a constant but varies according to the change in the energy of the body. The inertial mass of a system of bodies can even be regarded as a measure of its energy. The law of the conservation of the mass of a system becomes identical with the law of the conservation of energy, and is only valid provided that the system neither takes up nor sends out energy. Writing the expression for the energy in the form

物体的惯性质量不是一个常数，而是会根据其本身的能量变化的。一个系统的惯性质量甚至可以被视为其能量的量度。系统的质量守恒定律变得与能量守恒定律完全相同，但这只在当系统既不吸收也不发出能量时成立。从能量的下述表达形式

$$\frac{mc^2+E_0}{\sqrt{I-\frac{v^2}{c^2}}}$$

we see that the term mc^2, which has hitherto attracted our attention, is nothing else than the energy possessed by the body ** before it absorbed the energy E[0].

可以看出，我们前面见到的 mc^2 项，无非就是物体在吸收能量 E_0 之前本身所具有的能量**。

A direct comparison of this relation with experiment is not possible at the present time (1920; see *** Note, p. 48), owing to the fact that the changes in energy E[0] to which we can Subject a system are not large enough to make themselves perceptible as a change in the inertial mass of the system.

这种关系与实验的直接比较在目前是不可能的（1920 年，参见***第 48 页的注释），因为我们施于系统的能量 E_0 还不够大，所以产生的惯性质量的变化还看不出来。

$$\frac{E_0}{c^2}$$

is too small in comparison with the mass m, which was present before the alteration of the energy. It is owing to this circumstance that classical mechanics was able to establish successfully the conservation of mass as a law of independent validity.

与物体在能量变化之前就存在的质量 m 相比，$\frac{E_0}{c^2}$ 实在是太小了。正是由于这种原因，经典力学中的质量守恒才能作为一个独立有效的定律存在。

Let me add a final remark of a fundamental nature. The success of the Faraday-Maxwell interpretation of electromagnetic action at a distance resulted in physicists becoming convinced that there are no such things as instantaneous actions at a distance (not involving an intermediary medium) of the type of Newton's law of gravitation. According to the theory of relativity, action at a distance with the velocity of light always takes the place of instantaneous action at a distance or of action at a distance with an infinite velocity of transmission. This is connected with the fact that the velocity c plays a fundamental role in this theory. In Part II we shall see in what way this result becomes modified in the general theory of relativity.

还有最后一个有关根本的问题需要提及。法拉第-麦克斯韦对远距电磁作用的解释的成功，让物理学家们确信，不存在像牛顿万有引力定律那样的远距离瞬时作用（不涉及中间介质）。根据相对论，远距离作用总是以光速进行的，而不是远距离瞬时发生，或者说远距离作用不以无限大的速度传播。这是因为在相对论中，速度 c 扮演着一个（极限速度的）重要角色。在本文的第二部分，我们将看到在广义相对论中，这个结果会被如何修正。

Notes

*) E[0] is the energy taken up, as judged from a co-ordinate system moving with the body.

**) As judged from a co-ordinate system moving with the body.

***[Note] The equation E = mc^2 has been thoroughly proved time and again since this time.

备注

*）E_0是从与物体一起移动的坐标系得出的。

**）从与物体一起移动的坐标系得出的。

***[注] 从此以后，公式 $E=mc^2$ 已被一次又一次地证明了。

EXPERIENCE AND THE SPECIAL THEORY OF RELATIVITY

16. 经验与狭义相对论

To what extent is the special theory of relativity supported by experience? This question is not easily answered for the reason already mentioned in connection with the fundamental experiment of Fizeau. The special theory of relativity has crystallised out from the Maxwell-Lorentz theory of electromagnetic phenomena. Thus all facts of experience which support the electromagnetic theory also support the theory of relativity. As being of particular importance, I mention here the fact that the theory of relativity enables us to predict the effects produced on the light reaching us from the fixed stars. These results are obtained in an exceedingly simple manner, and the effects indicated, which are due to the relative motion of the earth with reference to those fixed stars are found to be in accord with experience. We refer to the yearly movement of the apparent position of the fixed stars resulting from the motion of the earth round the sun (aberration), and to the influence of the radial components of the relative motions of the fixed stars with respect to the earth on the colour of the light reaching us from them. The latter effect manifests itself in a slight displacement of the spectral lines of the light transmitted to us from a fixed star, as compared with the position of the same spectral lines when they are produced by a terrestrial source of light (Doppler principle). The experimental arguments in favour of the Maxwell-Lorentz theory, which are at the same time arguments in favour of the theory of relativity, are too numerous to be set forth here. In reality they limit the theoretical possibilities to such an extent, that no other theory than that of Maxwell and Lorentz has been able to hold its own when tested by experience.

狭义相对论在多大程度上得到了经验的支持？这个问题是不容易回答的，其原因已经在讲菲索的基本实验的时候提到过。狭义相对论是麦克斯韦-洛伦兹关于电磁现象的理论的结晶。因此所有支持电磁理论的事实也都支持相对论。由于其特别重要，我在这里要提到这个事实：相对论能够预测（运动）对来自恒星的光的影响。这些结果是用极其简单的方式得到的，并且，这些由地球相对于那些恒星的相对运动所引起的效果的预测与实际经验相符。这里指的是地球绕太阳运动所导致的恒星外观位置的年度变动（像差），以及恒星相对于地球的相对运动的径向分量对到达我们的光的颜色的影响。后者的影响体现在，与由地面光源产生的谱线相比，来自恒星的光谱线产生了轻微的位移（多普勒效应）。支持麦克斯韦-洛伦兹理论的实验数据，同时也就是支持相对论的的实验数据，实在是太多了，无法在这里一一列举。实际上，这些事实已经把理论上的可能性限制到了这样一种程度，也就是除了麦克斯韦和洛伦兹的理论之外，没有任何其它理论还能站得住脚。

But there are two classes of experimental facts hitherto obtained which can be represented in the Maxwell-Lorentz theory only by the introduction of an auxiliary hypothesis, which in itself -- i.e. without making use of the theory of relativity -- appears extraneous.

不过在迄今为止得到的实验事实中，有两类必须引入一个辅助假设，才能用麦克斯韦-洛伦兹理论来表示。这个辅助假设不需要相对论，但这似乎并无必要。

It is known that cathode rays and the so-called b-rays emitted by radioactive substances consist of negatively electrified particles (electrons) of very small inertia and large velocity. By examining the deflection of these rays under the influence of electric and magnetic fields, we can study the law of motion of these particles very exactly.

众所周知，阴极射线和放射性物质发出的所谓的β射线，是由带负电的粒子（电子）组成的，它的惯性很小但速度很大。通过检查这些射线在电磁场中的偏转，我们可以非常精确地研究这些粒子的运动规律。

In the theoretical treatment of these electrons, we are faced with the difficulty that electrodynamic theory of itself is unable to give an account of their nature. For since electrical masses of one sign repel each other, the negative electrical masses constituting the electron would necessarily be scattered under the influence of their mutual repulsions, unless there are forces of another kind operating between them, the nature of which has hitherto remained obscure to us.* If we now assume that the relative distances between the electrical masses constituting the electron remain unchanged during the motion of the electron (rigid connection in the sense of classical mechanics), we arrive at a law of motion of the electron which does not agree with experience. Guided by purely formal points of view, H. A. Lorentz was the first to introduce the hypothesis that the form of the electron experiences a contraction in the direction of motion in consequence of that motion. the contracted length being proportional to the expression

在理论上处理这些电子的时候，我们面临着这样的困难，既电动力学理论本身无法解释它们的性质。因为同样符号的电荷相互排斥，构成电子的负电荷必然会因为相互排斥而分散开，除非它们之间存在有另外一种迄今为止我们还不清楚的作用力。*如果我们假设构成电子的电荷之间的相对距离在运动过程中保持不变（经典力学意义上的刚性连接），我们就会得出一个与经验不符的电子运动定律。基于纯粹的形

式上的考虑，H.A.洛伦兹首先提出了这样的假设：电子束在运动方向上产生了收缩，收缩后的长度正比于如下表达式：

$$\sqrt{I - \frac{v^2}{c^2}}$$

This, hypothesis, which is not justifiable by any electrodynamical facts, supplies us then with that particular law of motion which has been confirmed with great precision in recent years.

这个假设并没有任何电动力学的事实根据，但它却为我们提供了一个特殊的运动定律，并且该定律近年来已经得到了相当精确的证实。

The theory of relativity leads to the same law of motion, without requiring any special hypothesis whatsoever as to the structure and the behaviour of the electron. We arrived at a similar conclusion in Section 13 in connection with the experiment of Fizeau, the result of which is foretold by the theory of relativity without the necessity of drawing on hypotheses as to the physical nature of the liquid.

相对论得出了相同的运动定律，但它不需要任何关于物质结构和电子行为的特殊假设。在第 13 节我们得出了一个类似的结论，该结论与菲索实验有关。相对论预测了菲索实验的结果，而无需关于对液体物理性质的假设。

The second class of facts to which we have alluded has reference to the question whether or not the motion of the earth in space can be made perceptible in terrestrial experiments. We have already remarked in Section 5 that all attempts of this nature led to a negative result. Before the theory of relativity was put forward, it was difficult to become reconciled to this negative result, for reasons now to be discussed. The inherited prejudices about time and space did not allow any doubt to arise as to the prime importance of the Galileian transformation for changing over from one body of reference to another. Now assuming that the Maxwell-Lorentz equations hold for a reference-body K, we then find that they do not hold for a reference-body K1 moving uniformly with respect to K, if we assume that the relations of the Galileian transformstion exist between the co-ordinates of K and K1. It thus appears that, of all Galileian co-ordinate systems, one (K) corresponding to a particular state of motion is physically unique. This result was interpreted physically by regarding K as at rest with respect to a hypothetical æther of space. On the other hand, all coordinate systems K1 moving relatively to K were to be regarded as in motion with respect to the æther. To this motion of K1 against the æther ("æther-drift " relative to K1) were attributed the more complicated laws which were supposed to hold relative to K1. Strictly speaking, such an æther-drift ought also to be assumed relative to the earth, and for a long time the efforts of physicists were devoted to attempts to detect the existence of an æther-drift at the earth's surface.

我们提到的第二类事实是关于这个问题的：地球在太空中的运动是否可以由在地球上的实验所探知。我们在第 5 节已经说过，所有这种性质的尝试都得出了负面的结果。在相对论提出之前，很难解释这个负面结果，下面我们将讨论其原因。由于一直以来对时间和空间的偏见，我们从不怀疑伽利略变换在参照系变换中的重要作用。假设伽利略变换关系对坐标系 K 和 K1 成立，如果麦克斯韦-洛伦兹方程在参照系 K 中成立，那么它在相对于 K 做匀速直线运动的 K1 中就不成立。因此看来，在所有伽利略坐标系，有一个坐标系（K）是独一无二的，它对应于某个特定的物理运动状态。这个结果被解释为 K 相对于所谓的以太空间处于静止状态，而所有其它相对于 K 运动的坐标系 K1 被视为在相对于以太在运动。由于 K1 相对于以太的运动（相对于 K1 的以太风），在 K1 中要使用更复杂的定律。严格来说，这种以太风相对于地球也应该存在，所以长期以来物理学家都在努力尝试探测这个相对于地球表面的以太风。

In one of the most notable of these attempts Michelson devised a method which appears as though it must be decisive. Imagine two mirrors so arranged on a rigid body that the reflecting surfaces face each other. A ray of light requires a perfectly definite time T to pass from one mirror to the other and back again, if the whole system be at rest with respect to the æther. It is found by calculation, however, that a slightly different time T1 is required for this process, if the body, together with the mirrors, be moving relatively to the æther. And yet another point: it is shown by calculation that for a given velocity v with reference to the æther, this time T1 is different when the body is moving perpendicularly to the planes of the mirrors from that resulting when the motion is parallel to these planes. Although the estimated difference between these two times is exceedingly small, Michelson and Morley performed an experiment involving interference in which this difference should have been clearly detectable. But the experiment gave a negative result -- a fact very perplexing to physicists. Lorentz and FitzGerald rescued the theory from this difficulty by assuming that the motion of the body relative to the æther produces a contraction of the body in the direction of motion, the amount of contraction being just sufficient to compensate for the differeace in time mentioned above. Comparison with the discussion in Section 11 shows that also from the standpoint of the theory of relativity this solution of the difficulty was the right one. But on the basis of the theory of relativity the method of interpretation is incomparably more satisfactory. According to this theory there is no such thing as a " specially favoured " (unique) co-ordinate system to occasion the introduction of the æther-idea, and hence there can be no æther-drift, nor any experiment with which to demonstrate it. Here the contraction of moving bodies follows from the two fundamental principles of the theory, without the introduction of particular hypotheses ; and as the prime factor involved in this contraction we find, not the motion in itself, to which we cannot attach any meaning, but the motion with respect to the body of reference chosen in the particular case in point. Thus for a co-ordinate system moving with the earth the mirror system of Michelson and Morley is not shortened, but it is shortened for a co-ordinate system which is at rest relatively to the sun.

在这些尝试中最著名的一个，是迈克尔逊设计的一个实验，它似乎能提供决定性的结论。想象在一个物体上装了彼此相对的两面镜子。如果整个系统相对于以太保持静止，那么一束光线从一个镜子到另一个镜子，然后再返回来，需要某一确定的时间 T。如果整个系统相对于以太在运动，我们可以算出，同样的过程需要稍微不同的时间 T1。我们还可以算出，同样是相对于以太以速度 v 运动，当运动垂直于镜面而不是平行于镜面时，所需的时间 T1 也不同。虽然按照我们的估计，这两个时间之间的差别非常小，但是它还是应该能被迈克尔逊和莫雷设计的干涉实验清晰地探测到。但实验给出了否定的结果，对此，物理学家感到非常困惑。为了解决这个问题，洛伦兹和菲茨杰拉德提出了这样一个假设：物体在相对于以太运动的方向上产生了收缩，并且该收缩量刚好能弥补上述的时间差。与第 11 节中的讨论相比较，从相对论的观点来看，这种解决办法是正确的。但是基于相对论的解释才是特别令人满意的。根据相对论，并不存在特别的（唯一的）坐标系用来引进以太这个概念，因此不可能有以太风，也没有任何实验可以用来展示它的存在。这里运动物体的收缩是从相对论的两个基本假定得出的，并不需要引入任何特别假设。这种收缩的主要因素，并不是运动本身，因为我们不能对其赋予任何意义，而是运动所参照的参考系。因此对于一个随地球运动的坐标系，迈克尔逊和莫雷的实验系统并没有缩短，但对于相对于太阳静止的坐标系来说，它缩短了。

Notes

*) The general theory of relativity renders it likely that the electrical masses of an electron are held together by gravitational forces.

注：
*) 广义相对论认为，电子的电荷可能是由引力保持在一起的。

MINKOWSKI'S FOUR-DIMENSIONAL SPACE

17. 闵可夫斯基的四维空间

The non-mathematician is seized by a mysterious shuddering when he hears of "four-dimensional" things, by a feeling not unlike that awakened by thoughts of the occult. And yet there is no more common-place statement than that the world in which we live is a four-dimensional space-time continuum.

当一个不是数学家的人听说叫做“四维”的东西的时候，他的身体就开始了一种神秘的颤抖，感觉就像是被玄秘的神灵所唤醒似的。然而再也找不到比这更普通的描述了：我们生活的世界就是一个四维的空间-时间连续体。

Space is a three-dimensional continuum. By this we mean that it is possible to describe the position of a point (at rest) by means of three numbers (co-ordinales) x, y, z, and that there is an indefinite number of points in the neighbourhood of this one, the position of which can be described by co-ordinates such as x[1], y[1], z[1], which may be as near as we choose to the respective values of the co-ordinates x, y, z, of the first point. In virtue of the latter property we speak of a " continuum," and owing to the fact that there are three co-ordinates we speak of it as being " three-dimensional."

空间是一个三维连续体。我们这么说的意思是，我们可以用三个数（坐标）x, y, z 来描述一个（静止的）点的位置。并且，在这个点的周围，存在无数个点，它们的位置可以用像 x[1]，y[1]，z[1]这样的坐标来描述，而这些坐标的值可以无限接近我们所选点的坐标值 x, y, z。这个性质，我们称其为“连续”，又因为有三个坐标，所以我们称其为“三维”。

Similarly, the world of physical phenomena which was briefly called " world " by Minkowski is naturally four dimensional in the space-time sense. For it is composed of individual events, each of which is described by four numbers, namely, three space co-ordinates x, y, z, and a time co-ordinate, the time value t. The" world" is in this sense also a continuum; for to every event there are as many "neighbouring" events (realised or at least thinkable) as we care to choose, the co-ordinates x[1], y[1], z[1], t[1] of which differ by an indefinitely small amount from those of the event x, y, z, t originally considered. That we have not been accustomed to regard the world in this sense as a four-dimensional continuum is due to the fact that in physics, before the advent of the theory of relativity, time played a different and more independent role, as compared with the space coordinates. It is for this reason that we have been in the habit of treating time as an independent continuum. As a matter of fact, according to classical mechanics, time is absolute, i.e. it is independent of the position and the condition of motion of the system of co-ordinates. We see this expressed in the last equation of the Galileian transformation (t1 = t)

与其类似，关于物理现象的世界（它被闵可夫斯基简称为“世界”），从空间—时间的意义上来说，自然就是四维的。因为它是由一个个单独的事件组成的，每个事件都由四个数字来描述，也就是三个空间坐标 x， y， z，和一个时间坐标，即时间值 t。 从这个意义上说，“世界”也是一个连续体。因为每一个事件都有想要多少就有多少的“邻居”事件，（这些事件是可以实现或至少是可以想象的），它们的坐标 x[1]， y[1]， z[1]， t[1] 和我们一开始选的事件的坐标可以有无限小的差值。我们之所以还没有习惯将这个意义上的世界视为四维连续体，是因为在物理学中，在相对论出现之前，与空间坐标相比，时间扮演了一个不同也更独立的角色。由于这个原因，我们一直习惯于把时间当作一个独立的连续体。事实上，根据经典力学，时间是绝对的，也就是它与位置无关，也与坐标系的运动状态无关。从伽利略变换的最后一个方程（t′ = t），我们可以看出这一点。

The four-dimensional mode of consideration of the "world" is natural on the theory of relativity, since according to this theory time is robbed of its independence. This is shown by the fourth equation of the Lorentz transformation:

在相对论中，把“世界”当作四维来考虑是很自然的。因为根据相对论，时间被剥夺了它的独立性。这点可以从洛伦兹变换的第四个方程看出来：

$$t' = \frac{t - \frac{v}{c^2}x}{\sqrt{I - \frac{v^2}{c^2}}}$$

Moreover, according to this equation the time difference Dt1 of two events with respect to K1 does not in general vanish, even when the time difference Dt1 of the same events with reference to K vanishes. Pure " space-distance " of two events with respect to K results in " time-distance " of the same events with respect to K. But the discovery of Minkowski, which was of importance for the formal development of the theory of relativity, does not lie here. It is to be found rather in the fact of his recognition that the four-dimensional space-time continuum of the theory of relativity, in its most essential formal properties, shows a pronounced relationship to the three-dimensional continuum of Euclidean geometrical space.* In order to give due prominence to this relationship, however, we must replace the usual time co-ordinate t by an imaginary magnitude eq. 25 proportional to it. Under these conditions, the natural laws satisfying the demands of the (special) theory of relativity assume mathematical forms, in which the time co-ordinate plays exactly the same role as the three space co-ordinates. Formally, these four co-ordinates correspond exactly to the three space co-ordinates in Euclidean geometry. It must be clear even to the non-mathematician that, as a consequence of this purely formal addition to our knowledge, the theory perforce gained clearness in no mean measure.

另外，根据这个方程，当两个事件相对于（坐标系）K 的时间差 dt 消失的时候，它们相对于（坐标系）K′ 的时间差 dt′ 通常也不会消失。两个事件相对于 K 的纯粹的“空间距离”产生了相同事件相对于 K′ 的“时间距离”。闵可夫斯基的发现对于相对论的正式推导是重要的，但它并不在此，而在于他认识到相对论的四维时空连续体，在它最基本的正式属性里，显示出了与欧几里得几何的三维连续空间的密切关系。* 不过为了突出这种关系，我们必须用一个与之成比例的虚数来代替通常的时间坐标 t（公式 25）。在这些条件下，满足（狭义）相对论要求的自然定律采取了这种数学形式，其中的时间坐标起的作用与三个空间坐标完全相同。正式地讲，这四个坐标正好对应欧几里得几何中的三个空间坐标。显然，即使你不是一个数学家也可以看出，由于这种纯粹形式上的补充，相对论就在很大程度上变清楚了。

These inadequate remarks can give the reader only a vague notion of the important idea contributed by Minkowski. Without it the general theory of relativity, of which the fundamental ideas are developed in the following pages, would perhaps have got no farther than its long clothes. Minkowski's work is doubtless difficult of access to anyone inexperienced in mathematics, but since it is not necessary to have a very exact grasp of this work in order to understand the fundamental ideas of either the special or the general theory of relativity, I shall leave it here at present, and revert to it only towards the end of Part 2.

对于闵可夫斯基的重要思想，这些大略的评论只能给读者一个模糊的概念。没有这些思想，接下来的对广义相对论的讨论，是走不了太远的。对于不熟悉数学的人来说，闵可夫斯基的文章无疑是难于理解的。但是因为无论是想了解狭义还是广义相对论的基本思想，都不需要非常准确地掌握闵可夫斯基的工作，对它的讨论我就到此为止了，等到接近第二部分的结尾时我们再返回来。

Notes

*) Cf. the somewhat more detailed discussion in Appendix II.

注：

*) 参见附录 II 中较为详细的讨论。

PART II

THE GENERAL THEORY OF RELATIVITY

SPECIAL AND GENERAL PRINCIPLE OF RELATIVITY

第二部分

广义相对论

狭义的和广义的相对性原理

The basal principle, which was the pivot of all our previous considerations, was the special principle of relativity, i.e. the principle of the physical relativity of all uniform motion. Let as once more analyse its meaning carefully.

It was at all times clear that, from the point of view of the idea it conveys to us, every motion must be considered only as a relative motion. Returning to the illustration we have frequently used of the embankment and the railway carriage, we can express the fact of the motion here taking place in the following two forms, both of which are equally justifiable :

(a) The carriage is in motion relative to the embankment,
(b) The embankment is in motion relative to the carriage.

In (a) the embankment, in (b) the carriage, serves as the body of reference in our statement of the motion taking place. If it is simply a question of detecting or of describing the motion involved, it is in principle immaterial to what reference-body we refer the motion. As already mentioned, this is self-evident, but it must not be confused with the much more comprehensive statement called "the principle of relativity," which we have taken as the basis of our investigations.

The principle we have made use of not only maintains that we may equally well choose the carriage or the embankment as our reference-body for the description of any event (for this, too, is self-evident). Our principle rather asserts what follows : If we formulate the general laws of nature as they are obtained from experience, by making use of

(a) the embankment as reference-body,
(b) the railway carriage as reference-body,

then these general laws of nature (e.g. the laws of mechanics or the law of the propagation of light in vacuo) have exactly the same form in both cases. This can also be expressed as follows : For the physical description of natural processes, neither of the reference bodies K, K1 is unique (lit. " specially marked out ") as compared with the other. Unlike the first, this latter statement need not of necessity hold a priori; it is not contained in the conceptions of " motion" and " reference-body " and derivable from them; only experience can decide as to its correctness or incorrectness.

Up to the present, however, we have by no means maintained the equivalence of all bodies of reference K in connection with the formulation of natural laws. Our course was more on the following

Iines. In the first place, we started out from the assumption that there exists a reference-body K, whose condition of motion is such that the Galileian law holds with respect to it : A particle left to itself and sufficiently far removed from all other particles moves uniformly in a straight line. With reference to K (Galileian reference-body) the laws of nature were to be as simple as possible. But in addition to K, all bodies of reference K1 should be given preference in this sense, and they should be exactly equivalent to K for the formulation of natural laws, provided that they are in a state of uniform rectilinear and non-rotary motion with respect to K ; all these bodies of reference are to be regarded as Galileian reference-bodies. The validity of the principle of relativity was assumed only for these reference-bodies, but not for others (e.g. those possessing motion of a different kind). In this sense we speak of the special principle of relativity, or special theory of relativity.

In contrast to this we wish to understand by the "general principle of relativity" the following statement : All bodies of reference K, K1, etc., are equivalent for the description of natural phenomena (formulation of the general laws of nature), whatever may be their state of motion. But before proceeding farther, it ought to be pointed out that this formulation must be replaced later by a more abstract one, for reasons which will become evident at a later stage.

Since the introduction of the special principle of relativity has been justified, every intellect which strives after generalisation must feel the temptation to venture the step towards the general principle of relativity. But a simple and apparently quite reliable consideration seems to suggest that, for the present at any rate, there is little hope of success in such an attempt; Let us imagine ourselves transferred to our old friend the railway carriage, which is travelling at a uniform rate. As long as it is moving unifromly, the occupant of the carriage is not sensible of its motion, and it is for this reason that he can without reluctance interpret the facts of the case as indicating that the carriage is at rest, but the embankment in motion. Moreover, according to the special principle of relativity, this interpretation is quite justified also from a physical point of view.

If the motion of the carriage is now changed into a non-uniform motion, as for instance by a powerful application of the brakes, then the occupant of the carriage experiences a correspondingly powerful jerk forwards. The retarded motion is manifested in the mechanical behaviour of bodies relative to the person in the railway carriage. The mechanical behaviour is different from that of the case previously considered, and for this reason it would appear to be impossible that the same mechanical laws hold relatively to the non-uniformly moving carriage, as hold with reference to the carriage when at rest or in uniform motion. At all events it is clear that the Galileian law does not hold with respect to the non-uniformly moving carriage. Because of this, we feel compelled at the present juncture to grant a kind of absolute physical reality to non-uniform motion, in opposition to the general principle of relatvity. But in what follows we shall soon see that this conclusion cannot be maintained.

THE GRAVITATIONAL FIELD

"If we pick up a stone and then let it go, why does it fall to the ground ?" The usual answer to this question is: "Because it is attracted by the earth." Modern physics formulates the answer rather differently for the following reason. As a result of the more careful study of electromagnetic phenomena, we have come to regard action at a distance as a process impossible without the intervention of some intermediary medium. If, for instance, a magnet attracts a piece of iron, we cannot be content to regard this as meaning that the magnet acts directly on the iron through the intermediate empty space, but we are constrained to imagine -- after the manner of Faraday -- that the magnet always calls into being something physically real in the space around it, that something being what we call a "magnetic field." In its turn this magnetic field operates on the piece of iron, so that the latter strives to move towards the magnet. We shall not discuss here the justification for this incidental conception, which is indeed a somewhat arbitrary one. We shall only mention that with its aid electromagnetic phenomena can be theoretically represented much more satisfactorily than without it, and this applies particularly to the transmission of electromagnetic waves. The effects of gravitation also are regarded in an analogous manner.

The action of the earth on the stone takes place indirectly. The earth produces in its surrounding a gravitational field, which acts on the stone and produces its motion of fall. As we know from experience, the intensity of the action on a body dimishes according to a quite definite law, as we proceed farther and farther away from the earth. From our point of view this means : The law governing the properties of the gravitational field in space must be a perfectly definite one, in order correctly to represent the diminution of gravitational action with the distance from operative bodies. It is something like this: The body (e.g. the earth) produces a field in its immediate neighbourhood directly; the intensity and direction of the field at points farther removed from the body are thence determined by the law which governs the properties in space of the gravitational fields themselves.

In contrast to electric and magnetic fields, the gravitational field exhibits a most remarkable property, which is of fundamental importance for what follows. Bodies which are moving under the sole influence of a gravitational field receive an acceleration, which does not in the least depend either on the material or on the physical state of the body. For instance, a piece of lead and a piece of wood fall in exactly the same manner in a gravitational field (in vacuo), when they start off from rest or with the same initial velocity. This law, which holds most accurately, can be expressed in a different form in the light of the following consideration.

According to Newton's law of motion, we have

(Force) = (inertial mass) x (acceleration),

where the "inertial mass" is a characteristic constant of the accelerated body. If now gravitation is the cause of the acceleration, we then have

(Force) = (gravitational mass) x (intensity of the gravitational field),

where the "gravitational mass" is likewise a characteristic constant for the body. From these two relations follows:

$$\text{(acceleration)} = \frac{\text{(gravitational mass)}}{\text{(inertial mass)}} \times \text{(intensity of the gravitational field).}$$

If now, as we find from experience, the acceleration is to be independent of the nature and the condition of the body and always the same for a given gravitational field, then the ratio of the gravitational to the inertial mass must likewise be the same for all bodies. By a suitable choice of units we can thus make this ratio equal to unity. We then have the following law: The gravitational mass of a body is equal to its inertial law.

It is true that this important law had hitherto been recorded in mechanics, but it had not been interpreted. A satisfactory interpretation can be obtained only if we recognise the following fact : The same quality of a body manifests itself according to circumstances as " inertia " or as " weight " (lit. " heaviness '). In the following section we shall show to what extent this is actually the case, and how this question is connected with the general postulate of relativity.

THE EQUALITY OF INERTIAL AND GRAVITATIONAL MASS AS AN ARGUMENT FOR THE GENERAL POSTULE OF RELATIVITY

We imagine a large portion of empty space, so far removed from stars and other appreciable masses, that we have before us approximately the conditions required by the fundamental law of Galilei. It is then possible to choose a Galileian reference-body for this part of space (world), relative to which points at rest remain at rest and points in motion continue permanently in uniform rectilinear motion. As reference-body let us imagine a spacious chest resembling a room with an observer inside who is equipped with apparatus. Gravitation naturally does not exist for this observer. He must fasten himself with strings to the floor, otherwise the slightest impact against the floor will cause him to rise slowly towards the ceiling of the room.

To the middle of the lid of the chest is fixed externally a hook with rope attached, and now a " being " (what kind of a being is immaterial to us) begins pulling at this with a constant force. The chest together with the observer then begin to move "upwards" with a uniformly accelerated motion. In course of time their velocity will reach unheard-of values -- provided that we are viewing all this from another reference-body which is not being pulled with a rope.

But how does the man in the chest regard the Process ? The acceleration of the chest will be transmitted to him by the reaction of the floor of the chest. He must therefore take up this pressure by means of his legs if he does not wish to be laid out full length on the floor. He is then standing in the chest in exactly the same way as anyone stands in a room of a home on our earth. If he releases a body which he previously had in his land, the accelertion of the chest will no longer be transmitted to this body, and for this reason the body will approach the floor of the chest with an accelerated relative motion. The observer will further convince himself that the acceleration of the body towards the floor of the chest is always of the same magnitude, whatever kind of body he may happen to use for the experiment.

Relying on his knowledge of the gravitational field (as it was discussed in the preceding section), the man in the chest will thus come to the conclusion that he and the chest are in a gravitational field which is constant with regard to time. Of course he will be puzzled for a moment as to why the chest does not fall in this gravitational field. just then, however, he discovers the hook in the middle of the lid of the chest and the rope which is attached to it, and he consequently comes to the conclusion that the chest is suspended at rest in the gravitational field.

Ought we to smile at the man and say that he errs in his conclusion ? I do not believe we ought to if we wish to remain consistent ; we must rather admit that his mode of grasping the situation violates neither reason nor known mechanical laws. Even though it is being accelerated with respect to the "Galileian space" first considered, we can nevertheless regard the chest as being at rest. We have thus good grounds for extending the principle of relativity to include bodies of reference which are accelerated with respect to each other, and as a result we have gained a powerful argument for a generalised postulate of relativity.

We must note carefully that the possibility of this mode of interpretation rests on the fundamental property of the gravitational field of giving all bodies the same acceleration, or, what comes to the same thing, on the law of the equality of inertial and gravitational mass. If this natural law did not exist, the man in the accelerated chest would not be able to interpret the behaviour of the bodies around him on the supposition of a gravitational field, and he would not be justified on the grounds of experience in supposing his reference-body to be " at rest."

Suppose that the man in the chest fixes a rope to the inner side of the lid, and that he attaches a body to the free end of the rope. The result of this will be to strech the rope so that it will hang " vertically " downwards. If we ask for an opinion of the cause of tension in the rope, the man in the chest will say: "The suspended body experiences a downward force in the gravitational field, and this is neutralised by the tension of the rope ; what determines the magnitude of the tension of the rope is the gravitational mass of the suspended body." On the other hand, an observer who is poised freely in space will interpret the condition of things thus : " The rope must perforce take part in the accelerated motion of the chest, and it transmits this motion to the body attached to it. The tension of the rope is just large enough to effect the acceleration of the body. That which determines the magnitude of the tension of the rope is the inertial mass of the body." Guided by this example, we see that our extension of the principle of relativity implies the necessity of the law of the equality of inertial and gravitational mass. Thus we have obtained a physical interpretation of this law.

From our consideration of the accelerated chest we see that a general theory of relativity must yield important results on the laws of gravitation. In point of fact, the systematic pursuit of the general idea of relativity has supplied the laws satisfied by the gravitational field. Before proceeding farther, however, I must warn the reader against a misconception suggested by these considerations. A gravitational field exists for the man in the chest, despite the fact that there was no such field for the co-ordinate system first chosen. Now we might easily suppose that the existence of a gravitational field is always only an apparent one. We might also think that, regardless of the kind of gravitational field which may be present, we could always choose another reference-body such that no gravitational field exists with reference to it. This is by no means true for all gravitational fields, but only for those of quite special form. It is, for instance, impossible to choose a body of reference such that, as judged from it, the gravitational field of the earth (in its entirety) vanishes.

We can now appreciate why that argument is not convincing, which we brought forward against the general principle of relativity at theend of Section 18. It is certainly true that the observer in the railway carriage experiences a jerk forwards as a result of the application of the brake, and that he recognises, in this the non-uniformity of motion (retardation) of the carriage. But he is compelled by nobody to refer this jerk to a " real " acceleration (retardation) of the carriage. He might also interpret his experience thus: " My body of reference (the carriage) remains permanently at rest. With reference to it, however, there exists (during the period of application of the brakes) a gravitational field which is directed forwards and which is variable with respect to time. Under the influence of this field, the embankment together with the earth moves non-uniformly in such a manner that their original velocity in the backwards direction is continuously reduced."

IN WHAT RESPECTS ARE THE FOUNDATIONS OF CLASSICAL MECHANICS AND OF THE SPECIAL THEORY OF RELATIVITY UNSATISFACTORY?

We have already stated several times that classical mechanics starts out from the following law: Material particles sufficiently far removed from other material particles continue to move uniformly in a straight line or continue in a state of rest. We have also repeatedly emphasised that this fundamental law can only be valid for bodies of reference K which possess certain unique states of motion, and which are in uniform translational motion relative to each other. Relative to other reference-bodies K the law is not valid. Both in classical mechanics and in the special theory of relativity we therefore differentiate between reference-bodies K relative to which the recognised " laws of nature " can be said to hold, and reference-bodies K relative to which these laws do not hold.

But no person whose mode of thought is logical can rest satisfied with this condition of things. He asks : " How does it come that certain reference-bodies (or their states of motion) are given priority over other reference-bodies (or their states of motion) ? What is the reason for this Preference? In order to show clearly what I mean by this question, I shall make use of a comparison.

I am standing in front of a gas range. Standing alongside of each other on the range are two pans so much alike that one may be mistaken for the other. Both are half full of water. I notice that steam is being emitted continuously from the one pan, but not from the other. I am surprised at this, even if I have never seen either a gas range or a pan before. But if I now notice a luminous something of bluish colour under the first pan but not under the other, I cease to be astonished, even if I have never before seen a gas flame. For I can only say that this bluish something will cause the emission of the steam, or at least possibly it may do so. If, however, I notice the bluish something in neither case, and if I observe that the one continuously emits steam whilst the other does not, then I shall remain astonished and dissatisfied until I have discovered some circumstance to which I can attribute the different behaviour of the two pans.

Analogously, I seek in vain for a real something in classical mechanics (or in the special theory of relativity) to which I can attribute the different behaviour of bodies considered with respect to the reference systems K and K1.* Newton saw this objection and attempted to invalidate it, but without success. But E. Mach recognsed it most clearly of all, and because of this objection he claimed that mechanics must be placed on a new basis. It can only be got rid of by means of a physics which is conformable to the general principle of relativity, since the equations of such a theory hold for every body of reference, whatever may be its state of motion.

Notes

*) The objection is of importance more especially when the state of motion of the reference-body is of such a nature that it does not require any external agency for its maintenance, e.g. in the case when the reference-body is rotating uniformly.

A FEW INFERENCES FROM THE GENERAL PRINCIPLE OF RELATIVITY

The considerations of Section 20 show that the general principle of relativity puts us in a position to derive properties of the gravitational field in a purely theoretical manner. Let us suppose, for instance, that we know the space-time " course " for any natural process whatsoever, as regards the manner in which it takes place in the Galileian domain relative to a Galileian body of reference K. By means of purely theoretical operations (i.e. simply by calculation) we are then able to find how this known natural process appears, as seen from a reference-body K1 which is accelerated relatively to K. But since a gravitational field exists with respect to this new body of reference K1, our consideration also teaches us how the gravitational field influences the process studied.

For example, we learn that a body which is in a state of uniform rectilinear motion with respect to K (in accordance with the law of Galilei) is executing an accelerated and in general curvilinear motion with respect to the accelerated reference-body K1 (chest). This acceleration or curvature corresponds to the influence on the moving body of the gravitational field prevailing relatively to K. It is known that a gravitational field influences the movement of bodies in this way, so that our consideration supplies us with nothing essentially new.

However, we obtain a new result of fundamental importance when we carry out the analogous consideration for a ray of light. With respect to the Galileian reference-body K, such a ray of light is transmitted rectilinearly with the velocity c. It can easily be shown that the path of the same ray of light is no longer a straight line when we consider it with reference to the accelerated chest (reference-body K1). From this we conclude, that, in general, rays of light are propagated curvilinearly in gravitational fields. In two respects this result is of great importance.

In the first place, it can be compared with the reality. Although a detailed examination of the question shows that the curvature of light rays required by the general theory of relativity is only exceedingly small for the gravitational fields at our disposal in practice, its estimated magnitude for light rays passing the sun at grazing incidence is nevertheless 1.7 seconds of arc. This ought to manifest itself in the following way. As seen from the earth, certain fixed stars appear to be in the neighbourhood of the sun, and are thus capable of observation during a total eclipse of the sun. At such times, these stars ought to appear to be displaced outwards from the sun by an amount indicated above, as compared with their apparent position in the sky when the sun is situated at another part of the heavens. The examination of the correctness or otherwise of this deduction is a problem of the greatest importance, the early solution of which is to be expected of astronomers.[2]*

In the second place our result shows that, according to the general theory of relativity, the law of the constancy of the velocity of light in vacuo, which constitutes one of the two fundamental assumptions in the special theory of relativity and to which we have already frequently referred, cannot claim any unlimited validity. A curvature of rays of light can only take place when the velocity of propagation of light varies with position. Now we might think that as a consequence of this, the special theory of relativity and with it the whole theory of relativity would be laid in the dust. But in reality this is not the case. We can only conclude that the special theory of relativity cannot claim an unlinlited domain of validity ; its results hold only so long as we are able to disregard the influences of gravitational fields on the phenomena (e.g. of light).

Since it has often been contended by opponents of the theory of relativity that the special theory of relativity is overthrown by the general theory of relativity, it is perhaps advisable to make the facts of the case clearer by means of an appropriate comparison. Before the development of electrodynamics the laws of electrostatics were looked upon as the laws of electricity. At the present time we know that electric fields can be derived correctly from electrostatic considerations only for the case, which is never strictly realised, in which the electrical masses are quite at rest relatively to each other, and to the co-ordinate system. Should we be justified in saying that for this reason electrostatics is overthrown by the field-equations of Maxwell in electrodynamics ? Not in the least. Electrostatics is contained in electrodynamics as a limiting case ; the laws of the latter lead directly to those of the former for the case in which the fields are invariable with regard to time. No fairer destiny could be allotted to any physical theory, than that it should of itself point out the way to the introduction of a more comprehensive theory, in which it lives on as a limiting case.

In the example of the transmission of light just dealt with, we have seen that the general theory of relativity enables us to derive theoretically the influence of a gravitational field on the course of natural processes, the Iaws of which are already known when a gravitational field is absent. But the most attractive problem, to the solution of which the general theory of relativity supplies the key, concerns the investigation of the laws satisfied by the gravitational field itself. Let us consider this for a moment.

We are acquainted with space-time domains which behave (approximately) in a " Galileian " fashion under suitable choice of reference-body, i.e. domains in which gravitational fields are absent. If we now refer such a domain to a reference-body K1 possessing any kind of motion, then relative to K1 there exists a gravitational field which is variable with respect to space and time.[3]** The character of this field will of course depend on the motion chosen for K1. According to the general theory of relativity, the general law of the gravitational field must be satisfied for all gravitational fields obtainable in this way. Even though by no means all gravitationial fields can be produced in this way, yet we may entertain the hope that the general law of gravitation will be derivable from such gravitational fields of a special kind. This hope has been realised in the most beautiful manner. But between the clear vision of this goal and its actual realisation it was necessary to surmount a serious difficulty, and as this lies deep at the root of things, I dare not withhold it from the reader. We require to extend our ideas of the space-time continuum still farther.

Notes

*) By means of the star photographs of two expeditions equipped by a Joint Committee of the Royal and Royal Astronomical Societies, the existence of the deflection of light demanded by theory was first confirmed during the solar eclipse of 29th May, 1919. (Cf. Appendix III.)

**) This follows from a generalisation of the discussion in Section 20

BEHAVIOUR OF CLOCKS AND MEASURING-RODS ON A ROTATING BODY OF REFERENCE

Hitherto I have purposely refrained from speaking about the physical interpretation of space- and time-data in the case of the general theory of relativity. As a consequence, I am guilty of a certain slovenliness of treatment, which, as we know from the special theory of relativity, is far from being unimportant and pardonable. It is now high time that we remedy this defect; but I would mention at the outset, that this matter lays no small claims on the patience and on the power of abstraction of the reader.

We start off again from quite special cases, which we have frequently used before. Let us consider a space time domain in which no gravitational field exists relative to a reference-body K whose state of motion has been suitably chosen. K is then a Galileian reference-body as regards the domain considered, and the results of the special theory of relativity hold relative to K. Let us supposse the same domain referred to a second body of reference K1, which is rotating uniformly with respect to K. In order to fix our ideas, we shall imagine K1 to be in the form of a plane circular disc, which rotates uniformly in its own plane about its centre. An observer who is sitting eccentrically on the disc K1 is sensible of a force which acts outwards in a radial direction, and which would be interpreted as an effect of inertia (centrifugal force) by an observer who was at rest with respect to the original reference-body K. But the observer on the disc may regard his disc as a reference-body which is " at rest " ; on the basis of the general principle of relativity he is justified in doing this. The force acting on himself, and in fact on all other bodies which are at rest relative to the disc, he regards as the effect of a gravitational field. Nevertheless, the space-distribution of this gravitational field is of a kind that would not be possible on Newton's theory of gravitation.* But since the observer believes in the general theory of relativity, this does not disturb him; he is quite in the right when he believes that a general law of gravitation can be formulated- a law which not only explains the motion of the stars correctly, but also the field of force experienced by himself.

The observer performs experiments on his circular disc with clocks and measuring-rods. In doing so, it is his intention to arrive at exact definitions for the signification of time- and space-data with reference to the circular disc K1, these definitions being based on his observations. What will be his experience in this enterprise ?

To start with, he places one of two identically constructed clocks at the centre of the circular disc, and the other on the edge of the disc, so that they are at rest relative to it. We now ask ourselves whether both clocks go at the same rate from the standpoint of the non-rotating Galileian reference-body K. As judged from this body, the clock at the centre of the disc has no velocity, whereas the clock at the edge of the disc is in motion relative to K in consequence of the rotation. According to a result obtained in Section 12, it follows that the latter clock goes at a rate permanently slower than that of the clock at the centre of the circular disc, i.e. as observed from K. It is obvious that the same effect would be noted by an observer whom we will imagine sitting alongside his clock at the centre of the circular disc. Thus on our circular disc, or, to make the case more general, in every gravitational field, a clock will go more quickly or less quickly, according to the position in which the clock is situated (at rest). For this reason it is not possible to obtain a reasonable definition of time with the aid of clocks which are arranged at rest with respect to the body of reference. A similar difficulty presents itself when we attempt to apply our earlier definition of simultaneity in such a case, but I do not wish to go any farther into this question.

Moreover, at this stage the definition of the space co-ordinates also presents insurmountable difficulties. If the observer applies his standard measuring-rod (a rod which is short as compared with the radius of the disc) tangentially to the edge of the disc, then, as judged from the Galileian system, the length of this rod will be less than I, since, according to Section 12, moving bodies suffer a shortening in the direction of the motion. On the other hand, the measaring-rod will not experience a shortening in length, as judged from K, if it is applied to the disc in the direction of the radius. If, then, the observer first measures the circumference of the disc with his measuring-rod and then the diameter of the disc, on dividing the one by the other, he will not obtain as quotient the familiar number p = 3.14 . . ., but a larger number,[4]** whereas of course, for a disc which is at rest with respect to K, this operation would yield p exactly. This proves that the propositions of Euclidean geometry cannot hold exactly on the rotating disc, nor in general in a gravitational field, at least if we attribute the length I to the rod in all positions and in every orientation. Hence the idea of a straight line also loses its meaning. We are therefore not in a position to define exactly the co-ordinates x, y, z relative to the disc by means of the method used in discussing the special theory, and as long as the co- ordinates and times of events have not been defined, we cannot assign an exact meaning to the natural laws in which these occur.

Thus all our previous conclusions based on general relativity would appear to be called in question. In reality we must make a subtle detour in order to be able to apply the postulate of general relativity exactly. I shall prepare the reader for this in the following paragraphs.

Notes

*) The field disappears at the centre of the disc and increases proportionally to the distance from the centre as we proceed outwards.

**) Throughout this consideration we have to use the Galileian (non-rotating) system K as reference-body, since we may only assume the validity of the results of the special theory of relativity relative to K (relative to K1 a gravitational field prevails).

EUCLIDEAN AND NON-EUCLIDEAN CONTINUUM

The surface of a marble table is spread out in front of me. I can get from any one point on this table to any other point by passing continuously from one point to a " neighbouring " one, and repeating this process a (large) number of times, or, in other words, by going from point to point without executing "jumps." I am sure the reader will appreciate with sufficient clearness what I mean here by " neighbouring " and by " jumps " (if he is not too pedantic). We express this property of the surface by describing the latter as a continuum.

Let us now imagine that a large number of little rods of equal length have been made, their lengths being small compared with the dimensions of the marble slab. When I say they are of equal length, I mean that one can be laid on any other without the ends overlapping. We next lay four of these little rods on the marble slab so that they constitute a quadrilateral figure (a square), the diagonals of which are equally long. To ensure the equality of the diagonals, we make use of a little testing-rod. To this square we add similar ones, each of which has one rod in common with the first. We proceed in like manner with each of these squares until finally the whole marble slab is laid out with squares. The arrangement is such, that each side of a square belongs to two squares and each corner to four squares.

It is a veritable wonder that we can carry out this business without getting into the greatest difficulties. We only need to think of the following. If at any moment three squares meet at a corner, then two sides of the fourth square are already laid, and, as a consequence, the arrangement of the remaining two sides of the square is already completely determined. But I am now no longer able to adjust the quadrilateral so that its diagonals may be equal. If they are equal of their own accord, then this is an especial favour of the marble slab and of the little rods, about which I can only be thankfully surprised. We must experience many such surprises if the construction is to be successful.

If everything has really gone smoothly, then I say that the points of the marble slab constitute a Euclidean continuum with respect to the little rod, which has been used as a " distance " (line-interval). By choosing one corner of a square as " origin" I can characterise every other corner of a square with reference to this origin by means of two numbers. I only need state how many rods I must pass over when, starting from the origin, I proceed towards the " right " and then " upwards," in order to arrive at the corner of the square under consideration. These two numbers are then the " Cartesian co-ordinates " of this corner with reference to the " Cartesian co-ordinate system" which is determined by the arrangement of little rods.

By making use of the following modification of this abstract experiment, we recognise that there must also be cases in which the experiment would be unsuccessful. We shall suppose that the rods " expand " by in amount proportional to the increase of temperature. We heat the central part of the marble slab, but not the periphery, in which case two of our little rods can still be brought into coincidence at every position on the table. But our construction of squares must necessarily come into disorder during the heating, because the little rods on the central region of the table expand, whereas those on the outer part do not.

With reference to our little rods -- defined as unit lengths -- the marble slab is no longer a Euclidean continuum, and we are also no longer in the position of defining Cartesian co-ordinates directly with their aid, since the above construction can no longer be carried out. But since there are other things which are not influenced in a similar manner to the little rods (or perhaps not at all) by the temperature of the table, it is possible quite naturally to maintain the point of view that the marble slab is a " Euclidean continuum." This can be done in a satisfactory manner by making a more subtle stipulation about the measurement or the comparison of lengths.

But if rods of every kind (i.e. of every material) were to behave in the same way as regards the influence of temperature when they are on the variably heated marble slab, and if we had no other means of detecting the effect of temperature than the geometrical behaviour of our rods in experiments analogous to the one described above, then our best plan would be to assign the distance one to two points on the slab, provided that the ends of one of our rods could be made to coincide with these two points ; for how else should we define the distance without our proceeding being in the highest measure grossly arbitrary ? The method of Cartesian coordinates must then be discarded, and replaced by another which does not assume the validity of Euclidean geometry for rigid bodies.* The reader will notice that the situation depicted here corresponds to the one brought about by the general postitlate of relativity (Section 23).

Notes

*) Mathematicians have been confronted with our problem in the following form. If we are given a surface (e.g. an ellipsoid) in Euclidean three-dimensional space, then there exists for this surface a two-dimensional geometry, just as much as for a plane surface. Gauss undertook the task of treating this two-dimensional geometry from first principles, without making use of the fact that the surface belongs to a Euclidean continuum of three dimensions. If we imagine constructions to be made with rigid rods in the surface (similar to that above with the marble slab), we should find that different laws hold for these from those resulting on the basis of Euclidean plane geometry. The surface is not a Euclidean continuum with respect to the rods, and we cannot define Cartesian co-ordinates in the surface. Gauss indicated the principles according to which we can treat the geometrical relationships in the surface, and thus pointed out the way to the method of Riemman of treating multi-dimensional, non-Euclidean continuum. Thus it is that mathematicians long ago solved the formal problems to which we are led by the general postulate of relativity.

GAUSSIAN CO-ORDINATES

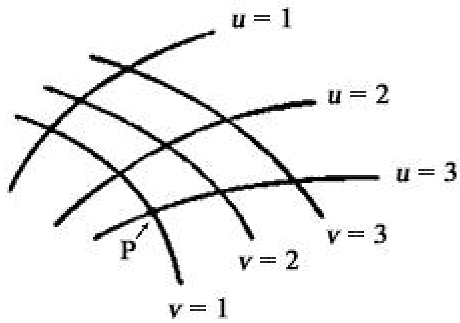

Fig. 4

According to Gauss, this combined analytical and geometrical mode of handling the problem can be arrived at in the following way. We imagine a system of arbitrary curves (see Fig. 4) drawn on the surface of the table. These we designate as u-curves, and we indicate each of them by means of a number. The Curves u= 1, u= 2 and u= 3 are drawn in the diagram. Between the curves u= 1 and u= 2 we must imagine an infinitely large number to be drawn, all of which correspond to real numbers lying between 1 and 2. fig. 04 We have then a system of u-curves, and this "infinitely dense" system covers the whole surface of the table. These u-curves must not intersect each other, and through each point of the surface one and only one curve must pass. Thus a perfectly definite value of u belongs to every point on the surface of the marble slab. In like manner we imagine a system of v-curves drawn on the surface. These satisfy the same conditions as the u-curves, they are provided with numbers in a corresponding manner, and they may likewise be of arbitrary shape. It follows that a value of u and a value of v belong to every point on the surface of the table. We call these two numbers the co-ordinates of the surface of the table (Gaussian co-ordinates). For example, the point P in the diagram has the Gaussian co-ordinates u= 3, v= 1. Two neighbouring points P and P1 on the surface then correspond to the co-ordinates

P: u,v

P1: u + du, v + dv,

where du and dv signify very small numbers. In a similar manner we may indicate the distance (line-interval) between P and P1, as measured with a little rod, by means of the very small number ds. Then according to Gauss we have

ds2 = g[11]du2 + 2g[12]dudv = g[22]dv2

where g[11], g[12], g[22], are magnitudes which depend in a perfectly definite way on u and v. The magnitudes g[11], g[12] and g[22], determine the behaviour of the rods relative to the u-curves and v-curves, and thus also relative to the surface of the table. For the case in which the points of the surface considered form a Euclidean continuum with reference to the measuring-rods, but only in this case, it is possible to draw the u-curves and v-curves and to attach numbers to them, in such a manner, that we simply have :

$$ds^2 = du^2 + dv^2$$

Under these conditions, the u-curves and v-curves are straight lines in the sense of Euclidean geometry, and they are perpendicular to each other. Here the Gaussian coordinates are samply Cartesian ones. It is clear that Gauss co-ordinates are nothing more than an association of two sets of numbers with the points of the surface considered, of such a nature that numerical values differing very slightly from each other are associated with neighbouring points " in space."

So far, these considerations hold for a continuum of two dimensions. But the Gaussian method can be applied also to a continuum of three, four or more dimensions. If, for instance, a continuum of four dimensions be supposed available, we may represent it in the following way. With every point of the continuum, we associate arbitrarily four numbers, x_1, x_2, x_3, x_4, which are known as " co-ordinates." Adjacent points correspond to adjacent values of the coordinates. If a distance ds is associated with the adjacent points P and P_1, this distance being measurable and well defined from a physical point of view, then the following formula holds:

$$ds^2 = g_{11}dx_1^2 + 2g_{12}dx_1dx_2 \ldots . g_{44}dx_4^2,$$

where the magnitudes g_{11}, etc., have values which vary with the position in the continuum. Only when the continuum is a Euclidean one is it possible to associate the co-ordinates $x_1 \ldots x_4$. with the points of the continuum so that we have simply

$$ds^2 = dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2.$$

In this case relations hold in the four-dimensional continuum which are analogous to those holding in our three-dimensional measurements.

However, the Gauss treatment for ds^2 which we have given above is not always possible. It is only possible when sufficiently small regions of the continuum under consideration may be regarded as Euclidean continua. For example, this obviously holds in the case of the marble slab of the table and local variation of temperature. The temperature is practically constant for a small part of the slab, and thus the geometrical behaviour of the rods is almost as it ought to be according to the rules of Euclidean geometry. Hence the imperfections of the construction of squares in the previous section do not show themselves clearly until this construction is extended over a considerable portion of the surface of the table.

We can sum this up as follows: Gauss invented a method for the mathematical treatment of continua in general, in which " size-relations " (" distances " between neighbouring points) are defined. To every point of a continuum are assigned as many numbers (Gaussian coordinates) as the continuum has dimensions. This is done in such a way, that only one meaning can be attached to the assignment, and that numbers (Gaussian coordinates) which differ by an indefinitely small amount are assigned to adjacent points. The Gaussian coordinate system is a logical generalisation of the Cartesian co-ordinate system. It is also applicable to non-Euclidean continua, but only when, with respect to the defined "size" or "distance," small parts of the continuum under consideration behave more nearly like a Euclidean system, the smaller the part of the continuum under our notice.

THE SPACE-TIME CONTINUUM OF THE SPEICAL THEORY OF RELATIVITY CONSIDERED AS A EUCLIDEAN CONTINUUM

We are now in a position to formulate more exactly the idea of Minkowski, which was only vaguely indicated in Section 17. In accordance with the special theory of relativity, certain co-ordinate systems are given preference for the description of the four-dimensional, space-time continuum. We called these " Galileian co-ordinate systems." For these systems, the four co-ordinates x, y, z, t, which determine an event or -- in other words, a point of the four-dimensional continuum -- are defined physically in a simple manner, as set forth in detail in the first part of this book. For the transition from one Galileian system to another, which is moving uniformly with reference to the first, the equations of the Lorentz transformation are valid. These last form the basis for the derivation of deductions from the special theory of relativity, and in themselves they are nothing more than the expression of the universal validity of the law of transmission of light for all Galileian systems of reference.

Minkowski found that the Lorentz transformations satisfy the following simple conditions. Let us consider two neighbouring events, the relative position of which in the four-dimensional continuum is given with respect to a Galileian reference-body K by the space co-ordinate differences dx, dy, dz and the time-difference dt. With reference to a second Galileian system we shall suppose that the corresponding differences for these two events are dx_1, dy_1, dz_1, dt_1. Then these magnitudes always fulfil the condition*

$$dx^2 + dy^2 + dz^2 - c^2dt^2 = dx_1{}^2 + dy_1{}^2 + dz_1{}^2 - c^2dt_1{}^2.$$

The validity of the Lorentz transformation follows from this condition. We can express this as follows: The magnitude

$$ds^2 = dx^2 + dy^2 + dz^2 - c^2dt^2,$$

which belongs to two adjacent points of the four-dimensional space-time continuum, has the same value for all selected (Galileian) reference-bodies. If we replace x, y, z, sq. rt. -I . ct , by x_1, x_2, x_3, x_4, we also obtaill the result that

$$ds^2 = dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2.$$

is independent of the choice of the body of reference. We call the magnitude ds the " distance " apart of the two events or four-dimensional points.

Thus, if we choose as time-variable the imaginary variable sq. rt. -I . ct instead of the real quantity t, we can regard the space-time contintium -- accordance with the special theory of relativity -- as a ", Euclidean " four-dimensional continuum, a result which follows from the considerations of the preceding section.

Notes

*) Cf. Appendixes I and 2. The relations which are derived there for the co-ordlnates themselves are valid also for co-ordinate differences, and thus also for co-ordinate differentials (indefinitely small differences).

THE SPACE-TIME CONTINUUM OF THE GENERAL THEORY OF REALTIIVTY IS NOT A ECULIDEAN CONTINUUM

In the first part of this book we were able to make use of space-time co-ordinates which allowed of a simple and direct physical interpretation, and which, according to Section 26, can be regarded as four-dimensional Cartesian co-ordinates. This was possible on the basis of the law of the constancy of the velocity of tight. But according to Section 21 the general theory of relativity cannot retain this law. On the contrary, we arrived at the result that according to this latter theory the velocity of light must always depend on the co-ordinates when a gravitational field is present. In connection with a specific illustration in Section 23, we found that the presence of a gravitational field invalidates the definition of the coordinates and the ifine, which led us to our objective in the special theory of relativity.

In view of the resuIts of these considerations we are led to the conviction that, according to the general principle of relativity, the space-time continuum cannot be regarded as a Euclidean one, but that here we have the general case, corresponding to the marble slab with local variations of temperature, and with which we made acquaintance as an example of a two-dimensional continuum. Just as it was there impossible to construct a Cartesian co-ordinate system from equal rods, so here it is impossible to build up a system (reference-body) from rigid bodies and clocks, which shall be of such a nature that measuring-rods and clocks, arranged rigidly with respect to one another, shaIll indicate position and time directly. Such was the essence of the difficulty with which we were confronted in Section 23.

But the considerations of Sections 25 and 26 show us the way to surmount this difficulty. We refer the fourdimensional space-time continuum in an arbitrary manner to Gauss co-ordinates. We assign to every point of the continuum (event) four numbers, x[1], x[2], x[3], x[4] (co-ordinates), which have not the least direct physical significance, but only serve the purpose of numbering the points of the continuum in a definite but arbitrary manner. This arrangement does not even need to be of such a kind that we must regard x[1], x[2], x[3], as "space" co-ordinates and x[4], as a " time " co-ordinate.

The reader may think that such a description of the world would be quite inadequate. What does it mean to assign to an event the particular co-ordinates x[1], x[2], x[3], x[4], if in themselves these co-ordinates have no significance ? More careful consideration shows, however, that this anxiety is unfounded. Let us consider, for instance, a material point with any kind of motion. If this point had only a momentary existence without duration, then it would to described in space-time by a single system of values x_1, x_2, x_3, x_4. Thus its permanent existence must be characterised by an infinitely large number of such systems of values, the co-ordinate values of which are so close together as to give continuity; corresponding to the material point, we thus have a (uni-dimensional) line in the four-dimensional continuum. In the same way, any such lines in our continuum correspond to many points in motion. The only statements having regard to these points which can claim a physical existence are in reality the statements about their encounters. In our mathematical treatment, such an encounter is expressed in the fact that the two lines which represent the motions of the points in question have a particular system of co-ordinate values, x_1, x_2, x_3, x_4, in common. After mature consideration the reader will doubtless admit that in reality such encounters constitute the only actual evidence of a time-space nature with which we meet in physical statements.

When we were describing the motion of a material point relative to a body of reference, we stated nothing more than the encounters of this point with particular points of the reference-body. We can also determine the corresponding values of the time by the observation of encounters of the body with clocks, in conjunction with the observation of the encounter of the hands of clocks with particular points on the dials. It is just the same in the case of space-measurements by means of measuring-rods, as a litttle consideration will show.

The following statements hold generally : Every physical description resolves itself into a number of statements, each of which refers to the space-time coincidence of two events A and B. In terms of Gaussian co-ordinates, every such statement is expressed by the agreement of their four co-ordinates x_1, x_2, x_3, x_4. Thus in reality, the description of the time-space continuum by means of Gauss co-ordinates completely replaces the description with the aid of a body of reference, without suffering from the defects of the latter mode of description; it is not tied down to the Euclidean character of the continuum which has to be represented.

EXACT FORMULATION OF THE GENERAL PRINCIPLE OF RELATIVITY

We are now in a position to replace the pro. visional formulation of the general principle of relativity given in Section 18 by an exact formulation. The form there used, "All bodies of reference K, K1, etc., are equivalent for the description of natural phenomena (formulation of the general laws of nature), whatever may be their state of motion," cannot be maintained, because the use of rigid reference-bodies, in the sense of the method followed in the special theory of relativity, is in general not possible in space-time description. The Gauss co-ordinate system has to take the place of the body of reference. The following statement corresponds to the fundamental idea of the general principle of relativity: "All Gaussian co-ordinate systems are essentially equivalent for the formulation of the general laws of nature."

We can state this general principle of relativity in still another form, which renders it yet more clearly intelligible than it is when in the form of the natural extension of the special principle of relativity. According to the special theory of relativity, the equations which express the general laws of nature pass over into equations of the same form when, by making use of the Lorentz transformation, we replace the space-time variables x, y, z, t, of a (Galileian) reference-body K by the space-time variables x1, y1, z1, t1, of a new reference-body K1. According to the general theory of relativity, on the other hand, by application of arbitrary substitutions of the Gauss variables x[1], x[2], x[3], x[4], the equations must pass over into equations of the same form; for every transformation (not only the Lorentz transformation) corresponds to the transition of one Gauss co-ordinate system into another.

If we desire to adhere to our "old-time" three-dimensional view of things, then we can characterise the development which is being undergone by the fundamental idea of the general theory of relativity as follows : The special theory of relativity has reference to Galileian domains, i.e. to those in which no gravitational field exists. In this connection a Galileian reference-body serves as body of reference, i.e. a rigid body the state of motion of which is so chosen that the Galileian law of the uniform rectilinear motion of "isolated" material points holds relatively to it.

Certain considerations suggest that we should refer the same Galileian domains to non-Galileian reference-bodies also. A gravitational field of a special kind is then present with respect to these bodies (cf. Sections 20 and 23).

In gravitational fields there are no such things as rigid bodies with Euclidean properties; thus the fictitious rigid body of reference is of no avail in the general theory of relativity. The motion of clocks is also influenced by gravitational fields, and in such a way that a physical definition of time which is made directly with the aid of clocks has by no means the same degree of plausibility as in the special theory of relativity.

For this reason non-rigid reference-bodies are used, which are as a whole not only moving in any way whatsoever, but which also suffer alterations in form ad lib. during their motion. Clocks, for which the law of motion is of any kind, however irregular, serve for the definition of time. We have to imagine each of these clocks fixed at a point on the non-rigid reference-body. These clocks satisfy only the one condition, that the "readings" which are observed simultaneously on adjacent clocks (in space) differ from each other by an indefinitely small amount. This non-rigid reference-body, which might appropriately be termed a "reference-mollusc", is in the main equivalent to a Gaussian four-dimensional co-ordinate system chosen arbitrarily. That which gives the "mollusc" a certain comprehensibility as compared with the Gauss co-ordinate system is the (really unjustified) formal retention of the separate existence of the space co-ordinates as opposed to the time co-ordinate. Every point on the mollusc is treated as a space-point, and every material point which is at rest relatively to it as at rest, so long as the mollusc is considered as reference-body. The general principle of relativity requires that all these molluscs can be used as reference-bodies with equal right and equal success in the formulation of the general laws of nature; the laws themselves must be quite independent of the choice of mollusc.

The great power possessed by the general principle of relativity lies in the comprehensive limitation which is imposed on the laws of nature in consequence of what we have seen above.

THE SOLUTION OF THE PROBLEM OF GRAVITATION ON THE BASIS OF THE GENERAL PRINCIPLE OF RELATIVITY

If the reader has followed all our previous considerations, he will have no further difficulty in understanding the methods leading to the solution of the problem of gravitation.

We start off on a consideration of a Galileian domain, i.e. a domain in which there is no gravitational field relative to the Galileian reference-body K. The behaviour of measuring-rods and clocks with reference to K is known from the special theory of relativity, likewise the behaviour of "isolated" material points; the latter move uniformly and in straight lines.

Now let us refer this domain to a random Gauss coordinate system or to a "mollusc" as reference-body K1. Then with respect to K1 there is a gravitational field G (of a particular kind). We learn the behaviour of measuring-rods and clocks and also of freely-moving material points with reference to K1 simply by mathematical transformation. We interpret this behaviour as the behaviour of measuring-rods, docks and material points tinder the influence of the gravitational field G. Hereupon we introduce a hypothesis: that the influence of the gravitational field on measuringrods, clocks and freely-moving material points continues to take place according to the same laws, even in the case where the prevailing gravitational field is not derivable from the Galfleian special care, simply by means of a transformation of co-ordinates.

The next step is to investigate the space-time behaviour of the gravitational field G, which was derived from the Galileian special case simply by transformation of the coordinates. This behaviour is formulated in a law, which is always valid, no matter how the reference-body (mollusc) used in the description may be chosen.

This law is not yet the general law of the gravitational field, since the gravitational field under consideration is of a special kind. In order to find out the general law-of-field of gravitation we still require to obtain a generalisation of the law as found above. This can be obtained without caprice, however, by taking into consideration the following demands:

(a) The required generalisation must likewise satisfy the general postulate of relativity.

(b) If there is any matter in the domain under consideration, only its inertial mass, and thus according to Section 15 only its energy is of importance for its etfect in exciting a field.

(c) Gravitational field and matter together must satisfy the law of the conservation of energy (and of impulse).

Finally, the general principle of relativity permits us to determine the influence of the gravitational field on the course of all those processes which take place according to known laws when a gravitational field is absent i.e. which have already been fitted into the frame of the special theory of relativity. In this connection we proceed in principle according to the method which has already been explained for measuring-rods, clocks and freely moving material points.

The theory of gravitation derived in this way from the general postulate of relativity excels not only in its beauty ; nor in removing the defect attaching to classical mechanics which was brought to light in Section 21; nor in interpreting the empirical law of the equality of inertial and gravitational mass ; but it has also already explained a result of observation in astronomy, against which classical mechanics is powerless.

If we confine the application of the theory to the case where the gravitational fields can be regarded as being weak, and in which all masses move with respect to the coordinate system with velocities which are small compared with the velocity of light, we then obtain as a first approximation the Newtonian theory. Thus the latter theory is obtained here without any particular assumption, whereas Newton had to introduce the hypothesis that the force of attraction between mutually attracting material points is inversely proportional to the square of the distance between them. If we increase the accuracy of the calculation, deviations from the theory of Newton make their appearance, practically all of which must nevertheless escape the test of observation owing to their smallness.

We must draw attention here to one of these deviations. According to Newton's theory, a planet moves round the sun in an ellipse, which would permanently maintain its position with respect to the fixed stars, if we could disregard the motion of the fixed stars themselves and the action of the other planets under consideration. Thus, if we correct the observed motion of the planets for these two influences, and if Newton's theory be strictly correct, we ought to obtain for the orbit of the planet an ellipse, which is fixed with reference to the fixed stars. This deduction, which can be tested with great accuracy, has been confirmed for all the planets save one, with the precision that is capable of being obtained by the delicacy of observation attainable at the present time. The sole exception is Mercury, the planet which lies nearest the sun. Since the time of Leverrier, it has been known that the ellipse corresponding to the orbit of Mercury, after it has been corrected for the influences mentioned above, is not stationary with respect to the fixed stars, but that it rotates exceedingly slowly in the plane of the orbit and in the sense of the orbital motion. The value obtained for this rotary movement of the orbital ellipse was 43 seconds of arc per century, an amount ensured to be correct to within a few seconds of arc. This effect can be explained by means of classical mechanics only on the assumption of hypotheses which have little probability, and which were devised solely for this purponse.

On the basis of the general theory of relativity, it is found that the ellipse of every planet round the sun must necessarily rotate in the manner indicated above ; that for all the planets, with the exception of Mercury, this rotation is too small to be detected with the delicacy of observation possible at the present time ; but that in the case of Mercury it must amount to 43 seconds of arc per century, a result which is strictly in agreement with observation.

Apart from this one, it has hitherto been possible to make only two deductions from the theory which admit of being tested by observation, to wit, the curvature of light rays by the gravitational field of the sun,*x and a displacement of the spectral lines of light reaching us from large stars, as compared with the corresponding lines for light produced in an analogous manner terrestrially (i.e. by the same kind of atom).** These two deductions from the theory have both been confirmed.

Notes

*) First observed by Eddington and others in 1919. (Cf. Appendix III, pp. 126-129).

**) Established by Adams in 1924. (Cf. p. 132)

PART III

CONSIDERATIONS ON THE UNIVERSE AS A WHOLE

COSMOLOGICAL DIFFICULTIES OF NEWTON'S THEORY

Part from the difficulty discussed in Section 21, there is a second fundamental difficulty attending classical celestial mechanics, which, to the best of my knowledge, was first discussed in detail by the astronomer Seeliger. If we ponder over the question as to how the universe, considered as a whole, is to be regarded, the first answer that suggests itself to us is surely this: As regards space (and time) the universe is infinite. There are stars everywhere, so that the density of matter, although very variable in detail, is nevertheless on the average everywhere the same. In other words: However far we might travel through space, we should find everywhere an attenuated swarm of fixed stars of approrimately the same kind and density.

This view is not in harmony with the theory of Newton. The latter theory rather requires that the universe should have a kind of centre in which the density of the stars is a maximum, and that as we proceed outwards from this centre the group-density of the stars should diminish, until finally, at great distances, it is succeeded by an infinite region of emptiness. The stellar universe ought to be a finite island in the infinite ocean of space.*

This conception is in itself not very satisfactory. It is still less satisfactory because it leads to the result that the light emitted by the stars and also individual stars of the stellar system are perpetually passing out into infinite space, never to return, and without ever again coming into interaction with other objects of nature. Such a finite material universe would be destined to become gradually but systematically impoverished.

In order to escape this dilemma, Seeliger suggested a modification of Newton's law, in which he assumes that for great distances the force of attraction between two masses diminishes more rapidly than would result from the inverse square law. In this way it is possible for the mean density of matter to be constant everywhere, even to infinity, without infinitely large gravitational fields being produced. We thus free ourselves from the distasteful conception that the material universe ought to possess something of the nature of a centre. Of course we purchase our emancipation from the fundamental difficulties mentioned, at the cost of a modification and complication of Newton's law which has neither empirical nor theoretical foundation. We can imagine innumerable laws which would serve the same purpose, without our being able to state a reason why one of them is to be preferred to the others ; for any one of these laws would be founded just as little on more general theoretical principles as is the law of Newton.

Notes

*) Proof -- According to the theory of Newton, the number of "lines of force" which come from infinity and terminate in a mass m is proportional to the mass m. If, on the average, the Mass density p[0] is constant throughout tithe universe, then a sphere of volume V will enclose the average man p[0]V. Thus the number of lines of force passing through the surface F of the sphere into its interior is proportional to p[0] V. For unit area of the surface of the sphere the

number of lines of force which enters the sphere is thus proportional to $p[0]$ V/F or to $p[0]R$. Hence the intensity of the field at the surface would ultimately become infinite with increasing radius R of the sphere, which is impossible.

THE POSSIBILITY OF A "FINITE" AND YET "UNBOUNDED" UNIVERSE

But speculations on the structure of the universe also move in quite another direction. The development of non-Euclidean geometry led to the recognition of the fact, that we can cast doubt on the infiniteness of our space without coming into conflict with the laws of thought or with experience (Riemann, Helmholtz). These questions have already been treated in detail and with unsurpassable lucidity by Helmholtz and Poincaré, whereas I can only touch on them briefly here.

In the first place, we imagine an existence in two dimensional space. Flat beings with flat implements, and in particular flat rigid measuring-rods, are free to move in a plane. For them nothing exists outside of this plane: that which they observe to happen to themselves and to their flat " things " is the all-inclusive reality of their plane. In particular, the constructions of plane Euclidean geometry can be carried out by means of the rods e.g. the lattice construction, considered in Section 24. In contrast to ours, the universe of these beings is two-dimensional; but, like ours, it extends to infinity. In their universe there is room for an infinite number of identical squares made up of rods, i.e. its volume (surface) is infinite. If these beings say their universe is " plane," there is sense in the statement, because they mean that they can perform the constructions of plane Euclidean geometry with their rods. In this connection the individual rods always represent the same distance, independently of their position.

Let us consider now a second two-dimensional existence, but this time on a spherical surface instead of on a plane. The flat beings with their measuring-rods and other objects fit exactly on this surface and they are unable to leave it. Their whole universe of observation extends exclusively over the surface of the sphere. Are these beings able to regard the geometry of their universe as being plane geometry and their rods withal as the realisation of " distance " ? They cannot do this. For if they attempt to realise a straight line, they will obtain a curve, which we " three-dimensional beings " designate as a great circle, i.e. a self-contained line of definite finite length, which can be measured up by means of a measuring-rod. Similarly, this universe has a finite area that can be compared with the area, of a square constructed with rods. The great charm resulting from this consideration lies in the recognition of the fact that the universe of these beings is finite and yet has no limits.

But the spherical-surface beings do not need to go on a world-tour in order to perceive that they are not living in a Euclidean universe. They can convince themselves of this on every part of their " world," provided they do not use too small a piece of it. Starting from a point, they draw " straight lines " (arcs of circles as judged in three dimensional space) of equal length in all directions. They will call the line joining the free ends of these lines a " circle." For a plane surface, the ratio of the circumference of a circle to its diameter, both lengths being measured with the same rod, is, according to Euclidean geometry of the plane, equal to a constant value p, which is independent of the diameter of the circle. On their spherical surface our flat beings would find for this ratio the value

$$\pi \frac{\sin\left(\frac{r}{R}\right)}{\left(\frac{r}{R}\right)}$$

i.e. a smaller value than p, the difference being the more considerable, the greater is the radius of the circle in comparison with the radius R of the " world-sphere." By means of this relation the spherical beings can determine the radius of their universe (" world "), even when only a relatively small part of their worldsphere is available for their measurements. But if this part is very small indeed, they will no longer be able to demonstrate that they are on a spherical " world " and not on a Euclidean plane, for a small part of a spherical surface differs only slightly from a piece of a plane of the same size.

Thus if the spherical surface beings are living on a planet of which the solar system occupies only a negligibly small part of the spherical universe, they have no means of determining whether they are living in a finite or in an infinite universe, because the " piece of universe " to which they have access is in both cases practically plane, or Euclidean. It follows directly from this discussion, that for our sphere-beings the circumference of a circle first increases with the radius until the " circumference of the universe " is reached, and that it thenceforward gradually decreases to zero for still further increasing values of the radius. During this process the area of the circle continues to increase more and more, until finally it becomes equal to the total area of the whole " world-sphere."

Perhaps the reader will wonder why we have placed our " beings " on a sphere rather than on another closed surface. But this choice has its justification in the fact that, of all closed surfaces, the sphere is unique in possessing the property that all points on it are equivalent. I admit that the ratio of the circumference c of a circle to its radius r depends on r, but for a given value of r it is the same for all points of the " worldsphere "; in other words, the " world-sphere " is a " surface of constant curvature."

To this two-dimensional sphere-universe there is a three-dimensional analogy, namely, the three-dimensional spherical space which was discovered by Riemann. its points are likewise all equivalent. It possesses a finite volume, which is determined by its "radius" (2p2R3). Is it possible to imagine a spherical space? To imagine a space means nothing else than that we imagine an epitome of our " space " experience, i.e. of experience that we can have in the movement of " rigid " bodies. In this sense we can imagine a spherical space.

Suppose we draw lines or stretch strings in all directions from a point, and mark off from each of these the distance r with a measuring-rod. All the free end-points of these lengths lie on a spherical surface. We can specially measure up the area (F) of this surface by means of a square made up of measuring-rods. If the universe is Euclidean, then F = 4pR2 ; if it is spherical, then F is always less than 4pR2. With increasing values of r, F increases from zero up to a maximum value which is determined by the " world-radius," but for still further increasing values of r, the area gradually diminishes to zero. At first, the straight lines which radiate from the starting point diverge farther and farther from one another, but later they approach each other, and finally they run together again at a "counter-point" to the starting point. Under such conditions they have traversed the whole spherical space. It is easily seen that the three-dimensional spherical space is quite analogous to the two-dimensional spherical surface. It is finite (i.e. of finite volume), and has no bounds.

It may be mentioned that there is yet another kind of curved space: " elliptical space." It can be regarded as a curved space in which the two " counter-points " are identical (indistinguishable from each other). An elliptical universe can thus be considered to some extent as a curved universe possessing central symmetry.

It follows from what has been said, that closed spaces without limits are conceivable. From amongst these, the spherical space (and the elliptical) excels in its simplicity, since all points on it are equivalent. As a result of this discussion, a most interesting question arises for astronomers and physicists, and that is whether the universe in which we live is infinite, or whether it is finite in the manner of the spherical universe. Our experience is far from being sufficient to enable us to answer this question. But the general theory of relativity permits of our answering it with a moduate degree of certainty, and in this connection the difficulty mentioned in Section 30 finds its solution.

THE STRUCTURE OF SPACE ACCORDING TO THE GENERAL THEORY OF RELATIVITY

According to the general theory of relativity, the geometrical properties of space are not independent, but they are determined by matter. Thus we can draw conclusions about the geometrical structure of the universe only if we base our considerations on the state of the matter as being something that is known. We know from experience that, for a suitably chosen co-ordinate system, the velocities of the stars are small as compared with the velocity of transmission of light. We can thus as a rough approximation arrive at a conclusion as to the nature of the universe as a whole, if we treat the matter as being at rest.

We already know from our previous discussion that the behaviour of measuring-rods and clocks is influenced by gravitational fields, i.e. by the distribution of matter. This in itself is sufficient to exclude the possibility of the exact validity of Euclidean geometry in our universe. But it is conceivable that our universe differs only slightly from a Euclidean one, and this notion seems all the more probable, since calculations show that the metrics of surrounding space is influenced only to an exceedingly small extent by masses even of the magnitude of our sun. We might imagine that, as regards geometry, our universe behaves analogously to a surface which is irregularly curved in its individual parts, but which nowhere departs appreciably from a plane: something like the rippled surface of a lake. Such a universe might fittingly be called a quasi-Euclidean universe. As regards its space it would be infinite. But calculation shows that in a quasi-Euclidean universe the average density of matter would necessarily be nil. Thus such a universe could not be inhabited by matter everywhere ; it would present to us that unsatisfactory picture which we portrayed in Section 30.

If we are to have in the universe an average density of matter which differs from zero, however small may be that difference, then the universe cannot be quasi-Euclidean. On the contrary, the results of calculation indicate that if matter be distributed uniformly, the universe would necessarily be spherical (or elliptical). Since in reality the detailed distribution of matter is not uniform, the real universe will deviate in individual parts from the spherical, i.e. the universe will be quasi-spherical. But it will be necessarily finite.
In fact, the theory supplies us with a simple connection * between the space-expanse of the universe and the average density of matter in it.

Notes

*) For the radius R of the universe we obtain the equation

$$R^2 = \frac{2}{Kp}$$

The use of the C.G.S. system in this equation gives 2/k = 1^.08.10^27; p is the average density of the matter and k is a constant connected with the Newtonian constant of gravitation.

APPENDIX I

SIMPLE DERIVATION OF THE LORENTZ TRANSFORMATION (SUPPLEMENTARY TO SECTION 11)

For the relative orientation of the co-ordinate systems indicated in Fig. 2, the x-axes of both systems pernumently coincide. In the present case we can divide the problem into parts by considering first only events which are localised on the x-axis. Any such event is represented with respect to the co-ordinate system K by the abscissa x and the time t, and with respect to the system K1 by the abscissa x' and the time t'. We require to find x' and t' when x and t are given.

A light-signal, which is proceeding along the positive axis of x, is transmitted according to the equation

$$x = ct$$

or

$$x - ct = 0 \quad . \quad . \quad . \quad (1).$$

Since the same light-signal has to be transmitted relative to K1 with the velocity c, the propagation relative to the system K1 will be represented by the analogous formula

$$x' - ct' = O \quad . \quad . \quad . \quad (2)$$

Those space-time points (events) which satisfy (x) must also satisfy (2). Obviously this will be the case when the relation

$$(x' - ct') = l\,(x - ct) \quad . \quad . \quad . \quad (3).$$

is fulfilled in general, where l indicates a constant ; for, according to (3), the disappearance of (x - ct) involves the disappearance of (x' - ct').

If we apply quite similar considerations to light rays which are being transmitted along the negative x-axis, we obtain the condition

$$(x' + ct') = \mu(x + ct) \quad . \quad . \quad . \quad (4).$$

By adding (or subtracting) equations (3) and (4), and introducing for convenience the constants a and b in place of the constants l and μ, where

$$a = \frac{\lambda + \mu}{2}$$

and

$$b = \frac{\lambda - \mu}{2}$$

we obtain the equations

$$\left.\begin{array}{l} x^1 = ax - bct \\ ct^1 = act - bx \end{array}\right\} \quad . \quad . \quad . \quad (5).$$

We should thus have the solution of our problem, if the constants a and b were known. These result from the following discussion.

For the origin of K1 we have permanently x' = 0, and hence according to the first of the equations (5)

$$x = \frac{bc}{a}t$$

If we call v the velocity with which the origin of K1 is moving relative to K, we then have

$$v = \frac{bc}{a} \qquad (6).$$

The same value v can be obtained from equations (5), if we calculate the velocity of another point of K1 relative to K, or the velocity (directed towards the negative x-axis) of a point of K with respect to K'. In short, we can designate v as the relative velocity of the two systems.

Furthermore, the principle of relativity teaches us that, as judged from K, the length of a unit measuring-rod which is at rest with reference to K1 must be exactly the same as the length, as judged from K', of a unit measuring-rod which is at rest relative to K. In order to see how the points of the x-axis appear as viewed from K, we only require to take a " snapshot " of K1 from K; this means that we have to insert a particular value of t (time of K), e.g. t = 0. For this value of t we then obtain from the first of the equations (5)

x' = ax

Two points of the x'-axis which are separated by the distance Dx' = I when measured in the K1 system are thus separated in our instantaneous photograph by the distance

$$\Delta x = \frac{I}{a} \quad . \quad . \quad . \quad (7).$$

But if the snapshot be taken from K'(t' = 0), and if we eliminate t from the equations (5), taking into account the expression (6), we obtain

$$x^1 = a\left(I - \frac{v^2}{c^2}\right)x$$

From this we conclude that two points on the x-axis separated by the distance I (relative to K) will be represented on our snapshot by the distance

$$\Delta x' = a\left(1 - \frac{v^2}{c^2}\right) \quad . \quad . \quad . \quad (7a).$$

But from what has been said, the two snapshots must be identical; hence Dx in (7) must be equal to Dx' in (7a), so that we obtain

$$a = \frac{1}{1 - \frac{v^2}{c^2}} \quad . \quad . \quad . \quad (7b).$$

The equations (6) and (7b) determine the constants a and b. By inserting the values of these constants in (5), we obtain the first and the fourth of the equations given in Section 11.

$$\left.\begin{aligned} x' &= \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \\ t' &= \frac{t - \frac{v}{c^2}x}{\sqrt{1 - \frac{v^2}{c^2}}} \end{aligned}\right\} \quad . \quad . \quad . \quad (8).$$

Thus we have obtained the Lorentz transformation for events on the x-axis. It satisfies the condition

x'2 - c^2t'2 = x2 - c^2t2 . . . (8a).

The extension of this result, to include events which take place outside the x-axis, is obtained by retaining equations (8) and supplementing them by the relations

$$\left.\begin{aligned} y' &= y \\ z' &= z \end{aligned}\right\} \quad . \quad . \quad . \quad (9).$$

In this way we satisfy the postulate of the constancy of the velocity of light in vacuo for rays of light of arbitrary direction, both for the system K and for the system K'. This may be shown in the following manner.

We suppose a light-signal sent out from the origin of K at the time t = 0. It will be propagated according to the equation

$$r = \sqrt{x^2 + y^2 + z^2} = ct$$

or, if we square this equation, according to the equation

$$x^2 + y^2 + z^2 = c^2t^2 = 0 \quad . \quad . \quad . \quad (10).$$

It is required by the law of propagation of light, in conjunction with the postulate of relativity, that the transmission of the signal in question should take place -- as judged from K1 -- in accordance with the corresponding formula

$$r' = ct'$$

or,

$$x'^2 + y'^2 + z'^2 - c^2t'^2 = 0 \quad . \quad . \quad . \quad (10a).$$

In order that equation (10a) may be a consequence of equation (10), we must have

$$x'^2 + y'^2 + z'^2 - c^2t'^2 = s\,(x^2 + y^2 + z^2 - c^2t^2) \qquad (11).$$

Since equation (8a) must hold for points on the x-axis, we thus have s = I. It is easily seen that the Lorentz transformation really satisfies equation (11) for s = I; for (11) is a consequence of (8a) and (9), and hence also of (8) and (9). We have thus derived the Lorentz transformation.

The Lorentz transformation represented by (8) and (9) still requires to be generalised. Obviously it is immaterial whether the axes of K1 be chosen so that they are spatially parallel to those of K. It is also not essential that the velocity of translation of K1 with respect to K should be in the direction of the x-axis. A simple consideration shows that we are able to construct the Lorentz transformation in this general sense from two kinds of transformations, viz. from Lorentz transformations in the special sense and from purely spatial transformations. which corresponds to the replacement of the rectangular co-ordinate system by a new system with its axes pointing in other directions.

Mathematically, we can characterise the generalised Lorentz transformation thus :

It expresses x', y', x', t', in terms of linear homogeneous functions of x, y, x, t, of such a kind that the relation

$$x'^2 + y'^2 + z'^2 - c^2t'^2 = x^2 + y^2 + z^2 - c^2t^2 \qquad (11a).$$

is satisficd identically. That is to say: If we substitute their expressions in x, y, x, t, in place of x', y', x', t', on the left-hand side, then the left-hand side of (11a) agrees with the right-hand side.

APPENDIX II

MINKOWSKI'S FOUR-DIMENSIONAL SPACE ("WORLD") (SUPPLEMENTARY TO SECTION 17)

We can characterise the Lorentz transformation still more simply if we introduce the imaginary eq. 25 in place of t, as time-variable. If, in accordance with this, we insert

$$x_1 = x$$
$$x_2 = y$$
$$x_3 = z$$
$$x_4 = \sqrt{-I}\cdot ct \quad \text{.......eq. 25}$$

and similarly for the accented system K1, then the condition which is identically satisfied by the transformation can be expressed thus :

$$x_1'^2 + x_2'^2 + x_3'^2 + x_4'^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2 \qquad (12).$$

That is, by the afore-mentioned choice of " coordinates," (11a) [see the end of Appendix II] is transformed into this equation.

We see from (12) that the imaginary time co-ordinate x_4, enters into the condition of transformation in exactly the same way as the space co-ordinates x_1, x_2, x_3. It is due to this fact that, according to the theory of relativity, the " time "x_4, enters into natural laws in the same form as the space co ordinates x_1, x_2, x_3.

A four-dimensional continuum described by the "co-ordinates" x_1, x_2, x_3, x_4, was called "world" by Minkowski, who also termed a point-event a " world-point." From a "happening" in three-dimensional space, physics becomes, as it were, an " existence " in the four-dimensional " world."

This four-dimensional " world " bears a close similarity to the three-dimensional " space " of (Euclidean) analytical geometry. If we introduce into the latter a new Cartesian co-ordinate system (x'_1, x'_2, x'_3) with the same origin, then x'_1, x'_2, x'_3, are linear homogeneous functions of x_1, x_2, x_3 which identically satisfy the equation

$$x'^2_1 + x'^2_2 + x'^2_3 = x^2_1 + x^2_2 + x^2_3$$

The analogy with (12) is a complete one. We can regard Minkowski's " world " in a formal manner as a four-dimensional Euclidean space (with an imaginary time coordinate) ; the Lorentz transformation corresponds to a " rotation " of the co-ordinate system in the fourdimensional " world."

APPENDIX III

THE EXPERIMENTAL CONFIRMATION OF THE GENERAL THEORY OF RELATIVITY

From a systematic theoretical point of view, we may imagine the process of evolution of an empirical science to be a continuous process of induction. Theories are evolved and are expressed in short compass as statements of a large number of individual observations in the form of empirical laws, from which the general laws can be ascertained by comparison. Regarded in this way, the development of a science bears some resemblance to the compilation of a classified catalogue. It is, as it were, a purely empirical enterprise.

But this point of view by no means embraces the whole of the actual process ; for it slurs over the important part played by intuition and deductive thought in the development of an exact science. As soon as a science has emerged from its initial stages, theoretical advances are no longer achieved merely by a process of arrangement. Guided by empirical data, the investigator rather develops a system of thought which, in general, is built up logically from a small number of fundamental assumptions, the so-called axioms. We call such a system of thought a theory. The theory finds the justification for its existence in the fact that it correlates a large number of single observations, and it is just here that the " truth " of the theory lies.

Corresponding to the same complex of empirical data, there may be several theories, which differ from one another to a considerable extent. But as regards the deductions from the theories which are capable of being tested, the agreement between the theories may be so complete that it becomes difficult to find any deductions in which the two theories differ from each other. As an example, a case of general interest is available in the province of biology, in the Darwinian theory of the development of species by selection in the struggle for existence, and in the theory of development which is based on the hypothesis of the hereditary transmission of acquired characters.

We have another instance of far-reaching agreement between the deductions from two theories in Newtonian mechanics on the one hand, and the general theory of relativity on the other. This agreement goes so far, that up to the preseat we have been able to find only a few deductions from the general theory of relativity which are capable of investigation, and to which the physics of pre-relativity days does not also lead, and this despite the profound difference in the fundamental assumptions of the two theories. In what follows, we shall again consider these important deductions, and we shall also discuss the empirical evidence appertaining to them which has hitherto been obtained.

(a) Motion of the Perihelion of Mercury

According to Newtonian mechanics and Newton's law of gravitation, a planet which is revolving round the sun would describe an ellipse round the latter, or, more correctly, round the common centre of gravity of the sun and the planet. In such a system, the sun, or the common centre of gravity, lies in one of the foci of the orbital ellipse in such a manner that, in the course of a planet-year, the distance sun-planet grows from a minimum to a maximum, and then decreases again to a minimum. If instead of Newton's law we insert a somewhat different law of attraction into the calculation, we find that, according to this new law, the motion would still take place in such a manner that the distance sun-planet exhibits periodic variations; but in this case the angle described by the line joining sun and planet during such a period (from perihelion--closest proximity to the sun--to perihelion) would differ from 360^0. The line of the orbit would not then be a closed one but in the course of time it would fill up an annular part of the orbital plane, viz. between the circle of least and the circle of greatest distance of the planet from the sun.

According also to the general theory of relativity, which differs of course from the theory of Newton, a small variation from the Newton-Kepler motion of a planet in its orbit should take place, and in such away, that the angle described by the radius sun-planet between one perhelion and the next should exceed that corresponding to one complete revolution by an amount given by

$$+ \frac{24 \pi^3 a^2}{T^2 c^2 (\mathrm{I} - e^2)}$$

(N.B. -- One complete revolution corresponds to the angle 2p in the absolute angular measure customary in physics, and the above expression giver the amount by which the radius sun-planet exceeds this angle during the interval between one perihelion and the next.) In this expression a represents the major semi-axis of the ellipse, e its eccentricity, c the velocity of light, and T the period of revolution of the planet. Our result may also be stated as follows : According to the general theory of relativity, the major axis of the ellipse rotates round the sun in the same sense as the orbital motion of the planet. Theory requires that this rotation should amount to 43 seconds of arc per century for the planet Mercury, but for the other Planets of our solar system its magnitude should be so small that it would necessarily escape detection. *

In point of fact, astronomers have found that the theory of Newton does not suffice to calculate the observed motion of Mercury with an exactness corresponding to that of the delicacy of observation attainable at the present time. After taking account of all the disturbing influences exerted on Mercury by the remaining planets, it was found (Leverrier: 1859; and Newcomb: 1895) that an unexplained perihelial movement of the orbit of Mercury remained over, the amount of which does not differ sensibly from the above mentioned +43 seconds of arc per century. The uncertainty of the empirical result amounts to a few seconds only.

(b) Deflection of Light by a Gravitational Field

In Section 22 it has been already mentioned that according to the general theory of relativity, a ray of light will experience a curvature of its path when passing through a gravitational field, this curvature being similar to that experienced by the path of a body which is projected through a gravitational field. As a result of this theory, we should expect that a ray of light which is passing close to a heavenly body would be deviated towards the latter. For a ray of light which passes the sun at a distance of D sun-radii from its centre, the angle of deflection (a) should amount to

$$a = \frac{1.7 \text{ seconds of arc}}{\Delta}$$

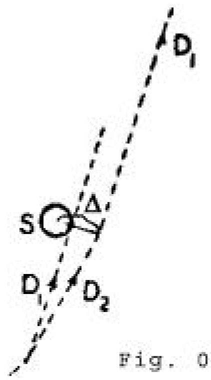

Fig. 05

It may be added that, according to the theory, half of Figure 05 this deflection is produced by the Newtonian field of attraction of the sun, and the other half by the geometrical modification (" curvature ") of space caused by the sun.

This result admits of an experimental test by means of the photographic registration of stars during a total eclipse of the sun. The only reason why we must wait for a total eclipse is because at every other time the atmosphere is so strongly illuminated by the light from the sun that the stars situated near the sun's disc are invisible. The predicted effect can be seen clearly from the accompanying diagram. If the sun (S) were not present, a star which is practically infinitely distant would be seen in the direction D[1], as observed front the earth. But as a consequence of the deflection of light from the star by the sun, the star will be seen in the direction D[2], i.e. at a somewhat greater distance from the centre of the sun than corresponds to its real position.

In practice, the question is tested in the following way. The stars in the neighbourhood of the sun are photographed during a solar eclipse. In addition, a second photograph of the same stars is taken when the sun is situated at another position in the sky, i.e. a few months earlier or later. As compared whh the standard photograph, the positions of the stars on the eclipse-photograph ought to appear displaced radially outwards (away from the centre of the sun) by an amount corresponding to the angle a.

We are indebted to the [British] Royal Society and to the Royal Astronomical Society for the investigation of this important deduction. Undaunted by the [first world] war and by difficulties of both a material and a psychological nature aroused by the war, these societies equipped two expeditions -- to Sobral (Brazil), and to the island of Principe (West Africa) -- and sent several of Britain's most celebrated astronomers (Eddington, Cottingham, Crommelin, Davidson), in order to obtain photographs of the solar eclipse of 29th May, 1919. The relative discrepancies to be expected between the stellar photographs obtained during the eclipse and the comparison photographs amounted to a few hundredths of a millimetre only. Thus great accuracy was necessary in making the adjustments required for the taking of the photographs, and in their subsequent measurement.

The results of the measurements confirmed the theory in a thoroughly satisfactory manner. The rectangular components of the observed and of the calculated deviations of the stars (in seconds of arc) are set forth in the following table of results :

Number of the Star.	First Co-ordinate.		Second Co-ordinate	
	Observed.	Calculated.	Observed.	Calculated.
11 . .	−0·19	−0·22	+0·16	+0·02
5 . .	+0·29	+0·31	−0·46	−0·43
4 . .	+0·11	+0·10	+0·83	+0·74
3 . .	+0·20	+0·12	+1·00	+0·87
6 . .	+0·10	+0·04	+0·57	+0·40
10 . .	−0·08	+0·09	+0·35	+0·32
2 . .	+0·95	+0·85	−0·27	−0·09

(c) Displacement of Spectral Lines Towards the Red

In Section 23 it has been shown that in a system K1 which is in rotation with regard to a Galileian system K, clocks of identical construction, and which are considered at rest with respect to the rotating reference-body, go at rates which are dependent on the positions of the clocks. We shall now examine this dependence quantitatively. A clock, which is situated at a distance r from the centre of the disc, has a velocity relative to K which is given by

$$V = wr$$

where w represents the angular velocity of rotation of the disc K1 with respect to K. If v[0], represents the number of ticks of the clock per unit time (" rate " of the clock) relative to K when the clock is at rest, then the " rate " of the clock (v) when it is moving relative to K with a velocity V, but at rest with respect to the disc, will, in accordance with Section 12, be given by

$$v = v_2 \sqrt{I - \frac{v^2}{c^2}}$$

or with sufficient accuracy by

$$v = v_0 \left(I - \frac{1}{2} \frac{v^2}{c^2} \right)$$

This expression may also be stated in the following form:

$$v = v_0 \left(I - \frac{I}{c^2} \frac{w^2 r^2}{2} \right)$$

If we represent the difference of potential of the centrifugal force between the position of the clock and the centre of the disc by f, i.e. the work, considered negatively, which must be performed on the unit of mass against the centrifugal force in order to transport it from the position of the clock on the rotating disc to the centre of the disc, then we have

$$\phi = \frac{w^2 r^2}{2}$$

From this it follows that

$$v = v_0 \left(I + \frac{\phi}{c^2} \right)$$

In the first place, we see from this expression that two clocks of identical construction will go at different rates when situated at different distances from the centre of the disc. This result is aiso valid from the standpoint of an observer who is rotating with the disc.

Now, as judged from the disc, the latter is in a gravititional field of potential f, hence the result we have obtained will hold quite generally for gravitational fields. Furthermore, we can regard an atom which is emitting spectral lines as a clock, so that the following statement will hold:

An atom absorbs or emits light of a frequency which is dependent on the potential of the gravitational field in which it is situated.

The frequency of an atom situated on the surface of a heavenly body will be somewhat less than the frequency of an atom of the same element which is situated in free space (or on the surface of a smaller celestial body).

Now f = - K (M/r), where K is Newton's constant of gravitation, and M is the mass of the heavenly body. Thus a displacement towards the red ought to take place for spectral lines produced at the surface of stars as compared with the spectral lines of the same element produced at the surface of the earth, the amount of this displacement being

$$\frac{\nu_0 - \nu}{\nu_0} = \frac{K}{c^2} \frac{M}{r}$$

For the sun, the displacement towards the red predicted by theory amounts to about two millionths of the wave-length. A trustworthy calculation is not possible in the case of the stars, because in general neither the mass M nor the radius r are known.

It is an open question whether or not this effect exists, and at the present time (1920) astronomers are working with great zeal towards the solution. Owing to the smallness of the effect in the case of the sun, it is difficult to form an opinion as to its existence. Whereas Grebe and Bachem (Bonn), as a result of their own measurements and those of Evershed and Schwarzschild on the cyanogen bands, have placed the existence of the effect almost beyond doubt, while other investigators, particularly St. John, have been led to the opposite opinion in consequence of their measurements.

Mean displacements of lines towards the less refrangible end of the spectrum are certainly revealed by statistical investigations of the fixed stars ; but up to the present the examination of the available data does not allow of any definite decision being arrived at, as to whether or not these displacements are to be referred in reality to the effect of gravitation. The results of observation have been collected together, and discussed in detail from the standpoint of the question which has been engaging our attention here, in a paper by E. Freundlich entitled "Zur Prüfung der allgemeinen Relativit&umlaut;ts-Theorie" (Die Naturwissenschaften, 1919, No. 35, p. 520: Julius Springer, Berlin).

At all events, a definite decision will be reached during the next few years. If the displacement of spectral lines towards the red by the gravitational potential does not exist, then the general theory of relativity will be untenable. On the other hand, if the cause of the displacement of spectral lines be definitely traced to the gravitational potential, then the study of this displacement will furnish us with important information as to the mass of the heavenly bodies. [5][A]

Notes

*) Especially since the next planet Venus has an orbit that is almost an exact circle, which makes it more difficult to locate the perihelion with precision.

The displacentent of spectral lines towards the red end of the spectrum was definitely established by Adams in 1924, by observations on the dense companion of Sirius, for which the effect is about thirty times greater than for the Sun. R.W.L. -- translator

APPENDIX IV

THE STRUCTURE OF SPACE ACCORDING TO THE GENERAL THEORY OF RELATIVITY
(SUPPLEMENTARY TO SECTION 32)

Since the publication of the first edition of this little book, our knowledge about the structure of space in the large (" cosmological problem ") has had an important development, which ought to be mentioned even in a popular presentation of the subject.

My original considerations on the subject were based on two hypotheses:

(1) There exists an average density of matter in the whole of space which is everywhere the same and different from zero.

(2) The magnitude (" radius ") of space is independent of time.

Both these hypotheses proved to be consistent, according to the general theory of relativity, but only after a hypothetical term was added to the field equations, a term which was not required by the theory as such nor did it seem natural from a theoretical point of view (" cosmological term of the field equations ").

Hypothesis (2) appeared unavoidable to me at the time, since I thought that one would get into bottomless speculations if one departed from it.

However, already in the 'twenties, the Russian mathematician Friedman showed that a different hypothesis was natural from a purely theoretical point of view. He realized that it was possible to preserve hypothesis (1) without introducing the less natural cosmological term into the field equations of gravitation, if one was ready to drop hypothesis (2). Namely, the original field equations admit a solution in which the " world radius " depends on time (expanding space). In that sense one can say, according to Friedman, that the theory demands an expansion of space.

A few years later Hubble showed, by a special investigation of the extra-galactic nebulae (" milky ways "), that the spectral lines emitted showed a red shift which increased regularly with the distance of the nebulae. This can be interpreted in regard to our present knowledge only in the sense of Doppler's principle, as an expansive motion of the system of stars in the large -- as required, according to Friedman, by the field equations of gravitation. Hubble's discovery can, therefore, be considered to some extent as a confirmation of the theory.

There does arise, however, a strange difficulty. The interpretation of the galactic line-shift discovered by Hubble as an expansion (which can hardly be doubted from a theoretical point of view), leads to an origin of this expansion which lies " only " about 10^9 years ago, while physical astronomy makes it appear likely that the development of individual stars and systems of stars takes considerably longer. It is in no way known how this incongruity is to be overcome.

I further want to remark that the theory of expanding space, together with the empirical data of astronomy, permit no decision to be reached about the finite or infinite character of (three-dimensional) space, while the original " static " hypothesis of space yielded the closure (finiteness) of space.

K = co-ordinate system
x, y = two-dimensional co-ordinates
x, y, z = three-dimensional co-ordinates
x, y, z, t = four-dimensional co-ordinates

t = time
I = distance
v = velocity

F = force
G = gravitational field

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